Experiments and modeling of anisotropic aluminum extrusions under multi-axial loading – Part II: Ductile fracture

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Abstract
The anisotropic ductile fracture of a 6260-T6 anisotropic aluminum alloy extrusion is investigated using a hybrid experimental–numerical approach. A basic fracture testing program covering a wide range of stress states and different material orientations is carried out. It comprises experiments on notched tensile specimens, tensile specimens with a central hole and butterfly shear specimens. The surface strain fields are obtained using two-dimensional Digital Image Correlation (DIC), while detailed finite element simulations are performed of all experiments to determine the local stress and strain histories inside the specimens. The analysis shows that the use of the newly-proposed extension of the Yld2000 yield function for three-dimensional stress states (see companion paper) together with an isotropic hardening law is able to predict the elasto-plastic behaviors of the present anisotropic aluminum alloy in all experiments. The experimental results show a strong dependency of the strain to fracture on the material orientation with respect to the loading direction. An uncoupled non-associated anisotropic fracture model is proposed which makes use of a stress state dependent weighting function and an anisotropic plastic strain measure. The latter is obtained from applying the von Mises equivalent plastic strain definition after the linear transformation of the plastic strain tensor. It is shown that the use of the isotropic Modified Mohr–Coulomb (MMC) stress state weighting function in this anisotropic fracture modeling framework provides accurate predictions of the onset of fracture for all thirteen fracture experiments.

1. Introduction

The prediction of anisotropic ductile fracture of metals is critical for the design of lightweight structures. This work focuses on the fracture of thin-walled aluminum extrusions which are used as primary load carrying members of car body structures. The formation of macroscopic cracks in metals is often considered as the result of the accumulation of damage within the material at the mesoscopic and/or microscopic level (Lemaitre, 1985). In the context of ductile fracture, damage is generally described as a sequence of events comprising void growth, nucleation and coalescence (McClintock, 1968; Rice and Tracey, 1969). Gurson (1977) proposed a simple porous plasticity model to account for the effect of void growth at the mesoscopic level on the effective plastic material response at the macroscopic level. Modifications introduced by Chu and Needleman (1980) and Tvergaard and Needleman (1984) made the model more complete by taking into consideration void nucleation and coalescence. Over the past two decades, numerous extensions have been made to the Gurson model (e.g. Huespe et al., 2009; Leblond et al., 1995; Lecarme et al., 2011; Nahshon and Hutchinson, 2008; Nielsen and Tvergaard,

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In addition to Gurson-like and CDM models, uncoupled phenomenological models have been developed. Neglecting the effect of damage on the elasto-plastic material behavior prior to fracture, one can utilize any standard metal plasticity model together with a separate phenomenological fracture model. It is usually postulated that fracture initiates when a weighted cumulative equivalent plastic strain reaches a critical value (e.g. Fischer et al., 1995). The weighting function depends on the Cauchy stress tensor and describes the effect of the stress state on the onset of fracture. Bao and Wierzbicki (2004) performed a comparative study on eight models of this type where different weighting functions have been formulated based on the respective work of McClintock (1968), Rice and Tracey (1969), LeRoy et al. (1981) and Clift et al. (1990) and the modified Cockcroft and Latham criterion (1968) by Oh et al. (1979). Another comparative study done by Zadpoor et al. (2009) covers the porous plasticity models, phenomenological models and the M–K model (Marciniak and Kuczynski, 1967) for sheet metal forming. Recently, Li et al. (2011) compared and evaluated the Gurson-like models, CDM models and the uncoupled phenomenological models with a series of tension and compression tests. Most early ductile fracture models use the stress triaxiality $\eta$ (ratio of hydrostatic stress to von Mises stress) as the only stress state parameter controlling fracture initiation. More recent studies (Coppola et al., 2009; Kim et al., 2004; Zhang et al., 2001) suggest that the ductility of metals also depends on the third stress invariant (Lode parameter). To formulate more general fracture models, Wilkins et al. (1980) and Xue (2007a,b) introduced the third stress invariant into the weighting function to account for the effect of both pressure and Lode parameter on ductile fracture. Gao et al., 2009, 2011 studied the effect of the hydrostatic stress and the third invariant of deviatoric stress tensor on both plasticity and ductile fracture. Bai and Wierzbicki (2010) formulated the so-called Modified Mohr–Coulomb (MMC) model which makes use of a stress triaxiality and Lode angle dependent weighting function that is obtained from transforming the stress-based Mohr–Coulomb failure criterion into the space of stress triaxiality, Lode parameter and equivalent plastic strain on the basis of an isotropic but stress-state dependent plasticity model (Bai and Wierzbicki, 2008). Assuming an isotropic fracture behavior, the MMC model has been applied for aluminum 6061-T6 sheets (Beese et al., 2010; Luo and Wierzbicki, 2009) and advanced high strength steels. Other approaches to predicting ductile fracture involve the modeling of the localization of deformation using theoretical bifurcation analysis (e.g. Li and Karr, 2009), micro-mechanics based analysis (e.g. Sun et al., 2009), and Forming Limit Curves (FLC).

One key challenge in modeling rolled/extruded aluminum sheets is that they exhibit significant anisotropy with regards to their plastic response and the initiation of fracture. The anisotropic plastic behavior of aluminum sheets has been extensively studied and is discussed in the accompanying Part I paper (Dunand et al., submitted for publication). Gurson-type of models have been extended to account for the effect of anisotropy on plasticity (Benzerza et al., 2004; Bron and Besson, 2006; Brunet et al., 2005; Rivalin et al., 2000), to describe the effect of non-spherical voids (Benzerza et al., 2004; Bron and Besson, 2006; Steglich et al., 2008), and their coalescence (Benzerza et al., 2002; Gologanu et al., 2001; Pardoen and Hutchinson, 2000, 2003). Monchiet et al. (2008) developed a homogenization-based macroscopic yield function which combines both orthotropic matrix and void shape effects. This function has been improved further by Keralavarma and Benzerza (2008) who considered richer deformation fields at the microlevel. In Keralavarma and Benzerza (2010), a more general case is addressed where the underlying micromechanical problem considers non-axisymmetric loading and spheroidal voids that are not aligned with the principal axes of matrix orthotropy. The reader is referred to Benzerza and Leblond (2010) for a comprehensive review of anisotropic void growth and coalescence models. Morgenev et al. (2009) proposed a new model which accounts for anisotropy throughout all stages of material damage. Based on the classical CDM models which employ a scalar measure of damage, more complex CDM models featuring a tensorial (and hence direction sensitive) damage variable have been proposed (e.g. Brüning et al., 2008; Brüning and Gerke, 2011; Chow and Wang, 1987; Chow et al., 2001; Hammi and Horstemeyer, 2011; Menzel and Steinmann, 2001; Voyiadjis and Dorgan, 2007).

In the present work, we propose a simple extension of uncoupled phenomenological fracture models to account for the effect of anisotropy on the initiation of macroscopic cracks in extruded aluminum. An extensive experimental program is carried out to investigate the effect of stress state and anisotropy on fracture. The local loading history up to the point of fracture is determined using a hybrid experimental–numerical approach. The results are used to calibrate and validate the proposed anisotropic fracture model over a wide range of stress states.

2. Fracture experiments

The experimental program is designed such that it provides insight in fracture initiation over a wide range of stress states. It includes experiments on flat tensile specimens with circular notches and specimens with a central hole. In addition, experiments are performed on a butterfly-shaped specimen for shear-dominated loading conditions. To investigate the possible anisotropy of the fracture properties, all specimens are extracted along three different orientations (0°, 45° and 90°) with respect to the material extrusion direction.
2.1. Material

All specimens in this work are prepared from the same batch of aluminum extrusions (see Fig. 1a). These extrusions of alloy AA6260-T6 had a wall thickness of about 2 mm. Preliminary experiments indicated that the fracture properties of the specimens depend on the location within the extruded profiles. To avoid this source of uncertainty in our experimental results, all fracture specimens are extracted from the same location within the extruded profiles. A full characterization of the elasto-plastic behavior of this alloy and its modeling is presented in the accompanying Part I paper (Dunand et al., submitted for publication). We recall that the plastic behavior of the present extruded aluminum is modeled using a standard plasticity model featuring: (1) the anisotropic non-quadratic Yld2000 (Barlat et al., 2003) yield function and its extension to three-dimensional stress states (Dunand et al., submitted for publication), (2) an associated flow rule, and (3) an isotropic hardening law.

2.2. Tensile specimens with circular notches

The first type of specimen employed in this work is a flat tensile specimen with a circular notch. The stress state within the gage section of this type of specimen depends on the notch radius \( R \). As \( R \) increases from zero to infinity, the stress state near the specimen center varies from plane strain tension to uniaxial tension; the corresponding stress triaxiality \( \eta \) ranges from about 0.33 to 0.58 (before localized necking). Three different notch radii are used in the present study: \( R = 20 \) mm, \( R = 10 \) mm and \( R = 5 \) mm (Fig. 1b). For each specimen, the notch is positioned such that the specimen width decreases from 20 mm in its shoulder region to 10 mm at the specimen center. A hydraulic testing machine (Instron Model 8080) with high pressure clamps is used to load these specimens under displacement control at a constant crosshead velocity of 0.4 mm/min. Digital Image Correlation (DIC) is used throughout all experiments to record the displacement field on both the front and
back surface of the specimen. A thin layer of white paint is applied to both surfaces along with black speckles (with an average speckle size of 70 μm); two digital cameras (QImaging Retiga 1300i with 105 mm Nikon Nikkor lenses) take about 250 pictures (resolution 1300 × 1028 pixels) throughout each experiment. The front camera is positioned at a distance of 1 m to take pictures of the entire specimen gage section (square pixel edge length of 39 μm); these pictures are used to calculate the displacement of the specimen boundaries. The second camera is positioned at a distance of only 0.25 m which resulted in a pixel edge length of 10 μm; these pictures are used to determine the local displacement field near the center of the specimen. The DIC software VIC2D (Correlated Solutions) is used to perform the DIC analysis. The logarithmic axial strain at the center of the gage section is also measured using a virtual extensometer (with an initial length $L_0 = 100$ pixels = 1 mm) located at the center of the specimen and aligned with the vertical axis of symmetry.

Eighteen notched tension experiments are performed. For each specimen geometry, three different material orientations are considered. Furthermore, each experiment is performed twice to confirm the repeatability of the experimental results. The subplots in Fig. 2 show the measured force–displacement curves for different notch radii. Different colors are used to differentiate among the material orientations: $0^\circ$ (blue), $45^\circ$ (red) and $90^\circ$ (black). The moments of the onset of fracture are highlighted by star symbols (the exact definition will be given in Section 3.1). A good repeatability is observed with regards to both the force–displacement responses and the displacement to fracture (variations of less than 3%). The experimental results show that the specimens for the transverse direction ($90^\circ$ with respect to extrusion direction) exhibit a lower displacement to fracture than those aligned with the extrusion and diagonal ($45^\circ$ with respect to extrusion direction) directions, irrespective of the notch radius. In other words, a noticeable and consistent anisotropy/directionality in ductility is observed for the present extruded aluminum sheet. As shown in Fig. 6, the strain field obtained through DIC exhibits significant strain localization near the center of the specimens which will be discussed in more detail in Section 3 along with corresponding numerical results.

1 For interpretation of colour in Figs. 2, 3, 5, 7, 9 and 13, the reader is referred to the web version of this article.
2.3. Tensile specimens with a central hole

In conventional uniaxial tension tests on dogbone specimens, the stress state within the gage section changes continuously from uniaxial tension to transverse plane strain tension due to necking prior to the onset of fracture. Here, tensile specimens with a central hole will be used as an alternative to dogbone specimens; Dunand and Mohr (2010) have shown that for this type of specimen the stress state remains more or less constant and close to uniaxial tension up to the point of crack initiation. As illustrated in Fig. 1b, the tensile specimens are 20 mm wide and have a 10 mm diameter hole at the center. The outer profiles of all specimens are cut using water jet machining. However, two different hole-cutting techniques are used: water jet cutting is used for the first set of specimens, while CNC milling (with a 0.125\(0.125\)\(0.125\) diameter end mill) is utilized for the second set to obtain a smoother edge finish. The experiments are carried out following the same procedure as for notched tensile specimens.

The measured force–displacement curves for all central-hole specimens are shown in Fig. 2d. Observe that the hole cutting procedure has a significant influence on the fracture initiation. For both 0° and 90° experiments, CNC-milled specimens feature a 25% higher fracture displacement than the water-jet-machined specimens. The same trend has been reported by Dunand and Mohr (2010) for TRIP steel, who pointed out that the rough edge with numerous geometric defects induced by the abrasive water-jet is responsible for the premature failure. Therefore, only the results for CNC milled specimens

![Fig. 3.](image-url)
are employed in the following analysis. As far as the anisotropy in ductility is concerned, the results for the central hole specimens indicate the same trend as those for notched tensile specimens: fracture occurs much earlier for when the loading axis is aligned with the extrusion direction than for the other two directions.

2.4. Butterfly specimens subject to shear loading

A butterfly-shaped specimen shown in Fig. 3a is used to perform fracture experiments for shear-dominated loading. The specimen features 2 mm thick shoulders and a 1 mm thick gage section. The thickness of the gage sections has been symmetrically reduced from both sheet surfaces through CNC milling. The shape of the specimen has been designed such that fracture initiates near the specimen center at a stress state of pure shear (Mohr and Henn, 2004, 2007; Dunand and Mohr, 2011). The horizontal actuator of a custom-made dual-actuator system (Mohr and Oswald, 2008) is used to apply a tangential displacement to the specimen boundary at a rate of 0.6 mm/min, while the vertical actuator is operated under force-control to guarantee zero vertical force throughout the experiment. As for the tensile experiments, two cameras are used to record the deformation of both the front and back surface of the butterfly specimen. The front camera captures the full view of the specimen gage section to monitor the onset of shear banding and fracture, while the back camera focuses on the center of the gage section to measure the displacement field with a higher spatial resolution. The engineering strain gage section to measure the displacement field with a higher spatial resolution.

The horizontal actuator of a custom-made dual-actuator system (Mohr and Henn, 2004, 2007; Dunand and Mohr, 2011) is therefore adopted to determine the stress and strain fields from numerical simulations of each experiment. In the present work, we define the instant of fracture initiation by the first visible crack in the paint layer on the specimen surface. In practice, we consider a displacement field discontinuity greater than 0.1 mm as a surface crack. Small star symbols highlight the corresponding moments of the onset of fracture in the measured force–displacement curves; the corresponding displacement (elongation of the specimen gage section) at that instant is defined as ‘displacement to fracture’, or ‘fracture displacement’. Note that the paint peels off prior to fracture in the shear experiments. Thus, the reported instants of onset of fracture in the shear experiments correspond to the instants of severe localization of deformation in shear bands. For this purpose, only one half of the butterfly specimen surface (w.r.t. the vertical central axis) is painted during shear testing, so that the acquired images of the unpainted specimen surface can be used to facilitate the detection of shear bands.

3. Stress-state and strain path prior to fracture

The stress and strain fields in all fracture experiments are non-uniform throughout the specimen gage section. The surface strain fields are determined experimentally from the DIC measurements. However, it is almost impossible to measure the stress fields as well as the strain fields inside the specimen. The hybrid experimental–numerical procedure proposed by Dunand and Mohr (2010) is therefore adopted to determine the stress and strain fields from numerical simulations of each experiment. In particular, we focus on the determination of the evolution of the stress triaxiality, the Lode angle parameter and the equivalent plastic strain ($\bar{\varepsilon}_p$) prior to the onset of fracture.

3.1. Definition of the onset of fracture

The identification of the onset of fracture in an experiment can be very challenging and lead to ambiguous results (e.g. Li and Wierzbicki, 2010; Luo and Wierzbicki, 2010). In the present work, we define the instant of fracture initiation by the first visible crack in the paint layer on the specimen surface. In practice, we consider a displacement field discontinuity greater than 0.1 mm as surface crack. Small star symbols highlight the corresponding moments of the onset of fracture in the measured force–displacement curves; the corresponding displacement (elongation of the specimen gage section) at that instant is defined as ‘displacement to fracture’, or ‘fracture displacement’. Note that the paint peels off prior to fracture in the shear experiments. Thus, the reported instants of onset of fracture in the shear experiments correspond to the instants of severe localization of deformation in shear bands. For this purpose, only one half of the butterfly specimen surface (w.r.t. the vertical central axis) is painted during shear testing, so that the acquired images of the unpainted specimen surface can be used to facilitate the detection of shear bands.

3.2. Details on the finite element models

Explicit finite element simulations are performed of all experiments (using the FE software package Abaqus). The specimens are discretized using reduced-integration eight-node solid elements (type C3D8R of the Abaqus element library). The anisotropic plasticity model (extension of the Yld2003 for general stress states) outlined in Part I (Dunand et al., submitted for publication) is implemented as a user material subroutine (VUMAT) and employed for all fracture simulations. Utilizing the symmetry of the specimen geometry and loading conditions, only one eighth of the tensile specimens and one half of the
shear specimen are modeled. During the simulations, a constant velocity is applied to the specimen boundaries; the loading velocity and material density are chosen such that at least 100,000 time steps are performed up to the instant of onset of fracture (quasi-static loading conditions). Special attention is paid to the following two points:

(1) Effect of mesh size: preliminary simulations are performed with four different mesh densities for notched tension with \( R = 10 \text{ mm} \). The meshes are made such that the elements at the center of the specimen have an aspect ratio of 1:1:1. After evaluating the mesh convergence with regards to the maximum equivalent plastic strain at the specimen center, we chose eight solid elements through the half-thickness for all our computations (the estimated strain changes by less than 1% when using 16 elements instead). Fig. 4a depicts the predicted force–displacement responses and strain history using different mesh densities, while Fig. 4b shows the selected mesh which corresponds to the ‘fine mesh’ (black curves) in Fig. 4a.

(2) Stress–strain curve extrapolation: For the present aluminum alloy, the strain hardening behavior can only be determined up to an equivalent plastic strain of 6% from a uniaxial tensile experiment (due to the onset of necking). An inverse approach is taken to extrapolate the hardening curve to large strains based on the experimental data for the \( R = 10 \text{ mm} \) notched tensile experiment (along the extrusion direction). As shown in Fig. 5, the widely-used Swift law extrapolation (black dashed line in Fig. 5a) overestimates the force–displacement curve (Fig. 5b), while the zero slope extension (black dots in Fig. 5a, using a zero slope curve beyond 6% strain) underestimates it. In an attempt to obtain a better prediction of the force–displacement response, a piece-wise linear extension (red line with squares in Fig. 5a) is utilized to represent the post-necking part of the hardening curve. The piecewise linear extension is optimized such that the simulation provides an accurate prediction (red curve in Fig. 5b) of the measured force–displacement curve (blue circles in Fig. 5b). In the following sections, the optimized piecewise linear hardening curve (red line with squares in Fig. 5a) is used in all simulations.

3.3. Simulation of notched tension

Nine simulations are carried out covering three distinct notch geometries (\( R = 20 \text{ mm}, 10 \text{ mm} \) and 5 mm) and three material orientations (\( \alpha = 0^\circ, 45^\circ \) and 90\(^\circ \)). Fig. 6 shows the axial strain (\( e_y \)) fields on the specimen surface measured by DIC and calculated by FEA, as well as at the specimen mid-plane provided by the FEA for all three notch geometries tested along the extrusion direction. Strain localization is observed in the center of the specimen on both the specimen surface (DIC) and the mid-plane (FEA). The same contour plots also demonstrate that the localization becomes more severe as the notch radius increases. The scale bars in Fig. 6 reveal that the strains on the specimen surface are significantly lower (about 50% lower) than those at the specimen mid-plane, and the surface strains given by DIC and FEA are of comparable magnitude. The contour plots for \( e_y \) on the axial and transverse cross-sections also show the strain gradient in thickness direction which is characteristic for localized necking. Differences between the DIC and FEA strain contours are partially due to fact that the DIC algorithm assumes that the specimen surface remains flat, while the FEA accounts for the development of a three-dimensional surface topology during necking; in addition, as a secondary effect, the reported FEA ‘surface’ strain is not exactly the surface strain, but the strain at the integration point of the surface elements (which is located at the center of these elements).

![Fig. 4. Mesh sensitivity study: (a) Effect of mesh density on the predicted force and local equivalent plastic strain versus displacement curves with a gage length of 30 mm. \( N_t \) represents the number of elements through half of the sheet thickness (1 mm) and (b) the selected mesh size for the present study, and it corresponds to the ‘fine mesh’ in the subplot a.](image-url)
The nine sub-plots in Fig. 7 demonstrate the good agreement of the experimental and computed force–displacement curves for all notched tensile experiments. In addition, the evolution of the logarithmic axial strain on the specimen surface obtained from FEA (black dashed line in Fig. 7) correlates well with that measured by DIC (green diamonds in Fig. 7). Assuming that fracture initiates in the mid-plane at the center of a notched specimen, the simulation results are used to extract the history of the stress triaxiality, the Lode angle parameter and the equivalent plastic strain.

3.4. Simulation of tension with a central hole

Following the same analysis procedure as for the notched tensile specimens, simulations are performed of the experiments on the specimens with a central hole for all material orientations ($\alpha = 0^\circ$, $45^\circ$ and $90^\circ$). As illustrated in Figs. 8a and b, the FE mesh corresponds to the upper-right eighth of the gage section. The DIC-measured and FEA-computed axial surface strain field is shown in Fig. 8c along with the computed strain distribution at the specimen mid-plane for a specimen loaded along the extrusion direction. Clearly, all strain contour plots in Fig. 8c show significant strain localization at the intersection of the hole with the transverse plane of symmetry. Similar to the results for the notched tensile specimens, the magnitude of the surface strain is about 50% lower than the computed strain at the mid-plane. As for the notched tensile experiments, the differences between the FEA and DIC strain contours are attributed to the in-plane deformation assumption of the DIC algorithm and differences in the location of the reported strains. Fig. 9 depicts the simulated force–displacement curves (red solid lines) along with the corresponding experimental data (blue dots). For each of the three material orientations, we observe a good agreement between the simulation and the experiment, which is also seen as a validation of the stress-strain curve extrapolation for large strains. The evolution of the equivalent plastic strain at the critical element (location of fracture initiation) is also shown in Fig. 9 (black solid lines).
3.5. Simulation of a butterfly specimen subject to shear loading

The FE mesh of the butterfly specimen is shown in Fig. 10a. Near the gage section center, the elements have the same size as those used for the notched tensile specimens. The shear experiments are simulated for all four material orientations $\alpha = 0^\circ, \pm 45^\circ$, and $90^\circ$. A constant velocity in the horizontal direction is uniformly imposed to the upper boundary, while the force is kept zero along the vertical direction. The simulated horizontal force versus the global engineering shear strain curves are in good agreement with the experimental data (Fig. 3b). For the material orientations $\alpha = \pm 45^\circ$, the predicted forces (blue and cyan dashed lines) are higher than the experimental measurements (blue and cyan solid lines) when the engineering strain exceeds 0.75. This discrepancy is attributed to the early formation of edge crack (red circles in Fig. 3e–f) during the experiments. The overall good agreement of simulations and experiments confirms the validity of the anisotropic plasticity model. Note that important differences with regards to the onset of shear band localization for different material orientations are successfully captured by the computational model (without using a failure criterion).

The computed contour plots of the equivalent plastic strain in the mid-plane at the onset of shear banding for $\alpha = 0^\circ$ and $\alpha = \pm 45^\circ$ are shown in Figs. 10b and c. The strain fields in the specimens with $\alpha = 90^\circ$ and $-45^\circ$ (not shown) are very similar to the respective fields for $\alpha = 0^\circ$ and $+45^\circ$. The star symbols in Fig. 3b indicate that the shear bands form at $\gamma = 0.17$ for the specimen with $\alpha = 0^\circ$ and at $\gamma = 1.09$ for specimens with $\alpha = \pm 45^\circ$. For $\alpha = 0^\circ$, two narrow shear bands initiate at the bound-
aries of the gage section, while the strain at the center of the specimen is much lower (by about 30%) than in the shear bands. The specimen with $\alpha = \pm 45^\circ$ can accommodate very large shear strains ($\gamma = 1.09$) before shear banding leads to fracture. No reliable DIC measurements are available for the shear experiments due to paint delamination and severe spatial gradients in the strain field. For the shear experiments with butterfly specimens, the center of the specimen (intersection of all three

Fig. 7. Comparison of experimental and simulation results in both force–displacement response and central logarithmic axial strain evolution for tensile specimen with circular cutouts. In the legend of these figures, 'FD' means force–displacement, 'PEEQ' denotes equivalent plastic strain, and 'log E' represents the logarithmic axial strain (gage length = 30 mm).
symmetry planes) is still regarded as the critical material point for pure shear loading, although the first cracks initiate at the boundaries of the gage section.

3.6. Summary: evolution of stress states and equivalent plastic strain

Fig. 11 shows the evolution of the equivalent plastic strain $\varepsilon_{pl}$ in terms of the stress triaxiality $\eta$ and the Lode angle parameter $\vartheta$ (see appendix for their mathematical definitions). The end points of the loading trajectories shown in Fig. 11 correspond to the experimentally-determined fracture initiation points indicated by the star symbols in Figs. 3b, 7 and 9. Those 13 experiments of interest include 12 tensile tests covering tension stress states in all three material orientations and one pure shear test on the butterfly specimen with $\alpha = \pm 45^\circ$. For the latter loading case, it is assumed that the FE computation of the strain and stress fields near the specimen center remain valid despite the early formation of small cracks near the specimen boundaries. The results from the shear tests with $\alpha = 0^\circ$, $90^\circ$, and $45^\circ$ are excluded since the direction dependency of the engineering strain to failure in these shear tests is already captured by the plasticity model (see Fig. 3b) even without a fracture criterion. Fig. 11b displays the histories of all 13 experiments in the space of $\varepsilon_{pl}$ and $\eta$. The results demonstrate that the equivalent plastic strain to fracture is highly material orientation dependent for the present extruded aluminum sheet, despite the fact that the evolution of $\eta$ is quite similar for the same specimen geometry in different material orientations. For instance, the central-hole specimens with $\alpha = 0^\circ$, $90^\circ$ and $-45^\circ$ feature almost identical trajectories of $\eta$ all the way to fracture, but the fracture strain for $\alpha = 0^\circ$ is much higher than that for $\alpha = 90^\circ$. As demonstrated by Figs. 11a and b, both $\eta$ and $\vartheta$ exhibit strong variations during loading; for example, in the case of notched tension with $R = 10$ mm, $\eta$ increases from 0.4 (before necking) to 0.92 at the onset of fracture (see Fig. 11b).

Table 1 summarizes the equivalent strain to fracture $\varepsilon_{fr}$ and the average values of the stress triaxiality and the Lode angle parameter during the entire loading process, defined as
In terms of the average values, the present experimental program covers the entire range of positive stress triaxialities for plane stress conditions \( (0 \leq \eta \leq 2/3) \) and a wide range of Lode angle parameters \( (-1/2 \leq \theta \leq 1) \).

4. Fracture modeling

An uncoupled phenomenological approach is employed to model the ductile fracture of the anisotropic AA6260-T6 sheets. In the case of isotropic materials, a damage indicator function is defined at each material point through a weighted von Mises equivalent plastic strain,

\[
\eta_{\text{arg}} = \frac{1}{\tilde{\varepsilon}_f} \int_0^{\tilde{\varepsilon}_f} \eta \, d\varepsilon_p \quad \text{and} \quad \tilde{\theta}_{\text{arg}} = \frac{1}{\tilde{\varepsilon}_f} \int_0^{\tilde{\varepsilon}_f} \tilde{\theta} \, d\varepsilon_p.
\]

In terms of the average values, the present experimental program covers the entire range of positive stress triaxialities for plane stress conditions \( (0 \leq \eta \leq 2/3) \) and a wide range of Lode angle parameters \( (-1/2 \leq \theta \leq 1) \).

\[
D = \int_0^{\tilde{\varepsilon}_f} w(\sigma) \, d\varepsilon_p,
\]

where \( w(\sigma) \) is an isotropic weighting function of the Cauchy stress tensor. Assuming an initial value of \( D = 0 \) for the virgin material, it is postulated that fracture initiates as \( D \) reaches unity. Simple anisotropic fracture models may be cast into this framework by substituting the isotropic von Mises equivalent plastic strain by an anisotropic equivalent plastic strain. In particular, when the anisotropic yield surface is written in terms of an anisotropic equivalent stress, it is natural to make use of the work-conjugate anisotropic equivalent plastic strain in the fracture model. In addition to this natural extension, we will also introduce a new equivalent plastic strain function which is obtained from the linear transformation of the plastic strain tensor. The wording “associated” is used for the first approach to emphasize that the equivalent plastic strain definition used in the fracture model is associated with the yield surface definition. Conversely, we make use of the wording “non-associated” to refer to the fracture model which employs an independent anisotropic equivalent plastic strain definition in the fracture model.

For both models, we make use of the Modified Mohr–Coulomb (MMC) weighting function \( w(\sigma) \) proposed by Bai and Wierzbicki (2010),

\[
w(\sigma) = \left\{ c_2 \left[ c_3 + \frac{\sqrt{3}}{2 - \sqrt{3}} (1 - c_1) \left( \sec \left( \frac{\tilde{\theta} \pi}{6} \right) - 1 \right) \right] \left[ \frac{1 + c_1^2}{3} \cos \left( \frac{\tilde{\theta} \pi}{6} \right) + \left( \eta + \frac{1}{3} \sin \left( \frac{\tilde{\theta} \pi}{6} \right) \right) \right] \right\}^{\frac{1}{3}}
\]
with the model parameters $c_1$, $c_2$ and $c_3$. Following the recommendation of Bai and Wierzbicki (2010), the exponent $n = 0.122$ is determined from an approximation of the stress–strain curve through Swift law (see Dunand et al. (submitted for publication) for details) and it is thus not treated as an independent model parameter. This particular three-parameter weighting function is used as it can be easily fitted to experimental data covering a wide range of stress states. It is noted that this weighting function is still isotropic which presents a strong simplifying assumption in our phenomenological modeling approach. The original Mohr–Coulomb (MC) failure model is a stress-based fracture criterion. Bai and Wierzbicki (2010) transformed this stress criterion into the space of $(\eta, \theta, \varepsilon_p)$ assuming a special Lode-angle dependent plasticity model.

4.1. Associated anisotropic fracture model

The Yld2000-3d yield function (Dunand et al., submitted for publication) is employed and hence the work-conjugate Yld2000-3d equivalent plastic strain is used in this fracture modeling approach. The fracture parameters $c_1$, $c_2$ and $c_3$ are calibrated through an inverse method using the hybrid experimental–numerical results. In particular, the fracture model parameters $[c_1, c_2, c_3]$ are calibrated such that the model predicts accurately the onset of fracture in all experiments where the major tensile strain is aligned with the extrusion direction (i.e. the data are shown in Fig. 11c and d). This is done by enforcing $D = 1$ at the instant of fracture initiation. Using the loading histories $\eta(t), \theta(t)$ and $\varepsilon_p(t)$ of the five calibration experiments (notched tension with three radii, tension with central hole and butterfly shearing) we obtain excellent agreement of the model with the experimental data for the extrusion direction. The calibrated model parameters of the associated MMC fracture model are given in Table 2.

Fig. 12 displays the underlying “fracture envelope” for the present associated anisotropic fracture model. It corresponds to a plot of the work-conjugate strain to fracture as a function of the stress triaxiality and the Lode angle parameter (i.e. the
Fig. 10. Numerical model of the butterfly specimen for the shear tests: (a) FE mesh of half (in thickness) of the butterfly specimen, (b) contour plot of the equivalent plastic strain at the onset of shear banding for $\alpha = 0^\circ$ ($\gamma = 0.17$) and (c) contour plot of the equivalent plastic strain at the onset of shear banding for $\alpha = +45^\circ$ ($\gamma = 1.49$).

Fig. 11. Loading paths at the critical material points of the specimens: (a) for all 13 experiments in the space of $(\bar{\epsilon}_p, \eta, \theta)$, (b) for all 13 experiments in the space of $(\bar{\epsilon}_p, \eta)$, (c) for the four tensile tests in the extrusion direction and one pure shear test ($\alpha = 45^\circ$) in the space of $(\bar{\epsilon}_p, \eta, \theta)$ and (d) for the four tensile tests in the extrusion direction and one pure shear test ($\alpha = 45^\circ$) in the space of $(\bar{\epsilon}_p, \eta)$. 
surface defined by $D = 1$) for the special case of monotonic proportional loading. The black lines depict the loading paths for the five experiments that have been used for calibration. Note that the loading path in our experiments is non-proportional and thus, the end points of the curves are not expected to lie on the visualized fracture envelope. Note that the fracture envelope cannot be used simply as a fracture strain limit (except for proportional loading); it must be interpreted as the reciprocal of the weighting function (Eq. (2)) for damage calculation, i.e. the lower the value of the fracture envelope, the faster damage accumulates for a given stress state.

We made use of the calibrated associated fracture model to predict the onset of fracture for all other experiments (i.e. the material orientations $90^\circ$ and $45^\circ$). Fig. 13a shows the predicted displacements to fracture as normalized by the corresponding experimental measurements. We observe good agreement for all $0^\circ$ experiments as these have been used for calibration. However, the calibrated model overestimates significantly the displacement to fracture for all other experiments. The largest relative difference is observed for the tensile test with central-hole specimens in transverse direction, where the predicted displacement to fracture is about 50% higher than that observed in the experiments. Fig. 13b shows the computed damage indicator $D$ at the experimentally measured instant of onset of fracture. This illustration elucidates that the use of the work-conjugate equivalent plastic strain underestimates the damage accumulation for the $90^\circ$ and $45^\circ$ material orientations.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Orientation (°)</th>
<th>$\bar{\varepsilon}_f$ (%)</th>
<th>$\eta_{avg}$ (%)</th>
<th>$\theta_{avg}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notched tensile specimen</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R = 5 \text{ mm}$</td>
<td>$0^\circ$</td>
<td>0.64</td>
<td>0.70</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>$45^\circ$</td>
<td>0.37</td>
<td>0.45</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>$90^\circ$</td>
<td>0.30</td>
<td>0.64</td>
<td>-0.06</td>
</tr>
<tr>
<td>$R = 10 \text{ mm}$</td>
<td>$0^\circ$</td>
<td>0.66</td>
<td>0.67</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>$45^\circ$</td>
<td>0.39</td>
<td>0.46</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>$90^\circ$</td>
<td>0.32</td>
<td>0.60</td>
<td>0.18</td>
</tr>
<tr>
<td>$R = 20 \text{ mm}$</td>
<td>$0^\circ$</td>
<td>0.69</td>
<td>0.64</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>$45^\circ$</td>
<td>0.43</td>
<td>0.45</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>$90^\circ$</td>
<td>0.36</td>
<td>0.56</td>
<td>0.40</td>
</tr>
<tr>
<td>Tensile specimen with a central hole</td>
<td>$\Phi = 10 \text{ mm}$</td>
<td>0.83</td>
<td>0.34</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>$0^\circ$</td>
<td>0.83</td>
<td>0.34</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>$45^\circ$</td>
<td>0.57</td>
<td>0.32</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>$90^\circ$</td>
<td>0.47</td>
<td>0.31</td>
<td>0.89</td>
</tr>
<tr>
<td>Butterfly shear specimen</td>
<td>$+45^\circ$</td>
<td>1.02</td>
<td>-0.05</td>
<td>-0.15</td>
</tr>
</tbody>
</table>

Table 2

Parameters for the associated anisotropic fracture model.

<table>
<thead>
<tr>
<th>$c_1$ (%)</th>
<th>$c_2$ (%)</th>
<th>$c_3$ (%)</th>
<th>LSE (–) (calibration tests)</th>
<th>LSE (–) (all tests)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.040</td>
<td>1.962</td>
<td>0.885</td>
<td>$3.0 \times 10^{-4}$</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Fig. 12. Fracture envelope for the associated anisotropic fracture model showing the work-conjugate equivalent strain to fracture as a function of the stress triaxiality and the Lode angle parameter. The black trajectories denote the loading path of the five experiments which are used for calibration, all featuring a maximum principal stress in the extrusion direction.
4.2. Non-associated anisotropic fracture model

An attempt is made to come up with a modified definition of the equivalent plastic strain to describe the anisotropic fracture properties with reasonable accuracy. Inspired by the formulation of anisotropic yield functions through the linear transformation of the stress tensor (Barlat et al., 2003; Cazacu and Barlat, 2001), we introduced an anisotropic equivalent plastic strain function which operates on a linearly transformed strain tensor. In vector notation, we write

\[
\mathbf{d}e_p = \sqrt{\frac{2}{3}} \mathbf{\beta} (d\mathbf{e}_p) \cdot (d\mathbf{e}_p) = \sqrt{\frac{2}{3}} \mathbf{\beta}^T (d\mathbf{e}_p) \cdot (d\mathbf{e}_p),
\]

where \(d\mathbf{e}_p\) denotes the increment in the plastic strain vector \(\mathbf{e}_p = (\varepsilon_{11}^p, \varepsilon_{22}^p, \varepsilon_{33}^p, \sqrt{2} \varepsilon_{12}^p, \sqrt{2} \varepsilon_{13}^p, \sqrt{2} \varepsilon_{23}^p)\), while \(\mathbf{\beta}^T \mathbf{\beta}\) is a positive semi-definite matrix which characterizes the linear transformation of the strain vector. Note that for \(\mathbf{\beta} = 1\), the above definition corresponds to the von Mises equivalent plastic strain.

In the present work, we limit our attention to a diagonal transformation matrix with only three independent coefficients,

\[
\mathbf{\beta} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_{22} & 0 & 0 & 0 & 0 \\
1 & 0 & \beta_{33} & 0 & 0 & 0 \\
1 & 0 & 0 & \beta_{12} & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

We assume \(\beta_{23} = \beta_{31} = 1\) as none of our experiments comprises substantial out-of-plane loading. Furthermore, as for the previous associated fracture model, we will make use of the isotropic MMC weighting function to account for the effect of stress.

![Fig. 13. Evaluation of the existing associated model with MMC fracture envelope and isotropic damage rule based on work-conjugate equivalent plastic strain: (a) predicted damage values at the experimentally-measured onset of fracture, (b) ratio of predicted to measured fracture displacement. Evaluation of the new non-associated model with MMC fracture envelope and anisotropic damage rule based on a linear transformed equivalent strain, (c) predicted damage values at the experimentally-measured onset of fracture and (d) ratio of predicted to measured fracture displacement. The red rectangular frames indicate the tests used for calibration of the non-associated model.](image-url)
As a result, six parameters need to be calibrated from experiments: three parameters associated with the linear-transformation of the strain vector and three parameters of the MMC weighting function. Seven experiments are used to calibrate the model parameters \(c_1, c_2, c_3, b_{22}, b_{33}, b_{12}\): the notched tensile tests with \(R = 10\) mm for all three material orientations, the tensile tests with central hole for all three material orientations, and the shear test with \(\alpha = +45^\circ\). The determination of the corresponding Least Squared Error (LSE) of damage predictions requires the history of the stress triaxiality, the Lode angle parameter and the plastic strain tensor. After minimization we obtain an LSE of \(3.98 \times 10^{-3}\), which indicates a good fit of the model to the experimental data for the calibration experiment. The corresponding fracture model parameters are given in Table 3.

To validate the proposed non-associated anisotropic fracture model, we compute the displacement at the onset of fracture for all experiments. The summary plot in Fig. 13c shows that proposed model provides good predictions of the displacement to fracture for all thirteen experiments. The biggest relative error is 8.5%; it is observed for a notched tensile test with \(R = 20\) mm at a material orientation of \(90^\circ\). Evaluation of the damage indicator at the experimentally-measured instant of onset of fracture (Fig. 13d) confirms the good agreement of the experiments and simulations. Moreover, as illustrated by Tables 2 and 3, the accumulated least square error of all 13 experiments with the new model is significantly lower than that with the associated model.

In the case of the associated fracture model, the work-conjugate equivalent plastic strain to fracture for monotonic proportional loading is a function of the normalized stress invariants only (Fig. 12). However, in the case of the non-associated fracture model, the work-conjugate equivalent plastic strain to fracture depends also on the direction of loading with respect to the material axes. To illustrate this direction dependency, selected fracture envelopes are computed assuming (1) the principal axes of the Cauchy stress tensor to be aligned with the material coordinate system, and (2) the minimum principal stress to be aligned with the sheet thickness direction. In particular, we consider the following three special cases:

1. Maximum principal stress along the \(0^\circ\)-direction.
2. Maximum principal stress along the \(90^\circ\)-direction.
3. Maximum principal stress along the \(45^\circ\)-direction.

Fig. 14 shows the corresponding fracture envelopes in the space of \((\eta, \tilde{\eta}, \tilde{\eta}_t)\). Analogously to the plot shown for the associated fracture model, the vertical coordinate may be interpreted as a measure for the damage accumulation. For example, the surfaces for the \(90^\circ\)-direction and \(45^\circ\)-direction lie well below the surface for \(0^\circ\)-direction which implies that damage accumulates much faster when the maximum principal stress is aligned with those two directions as compared to the \(0^\circ\)-direction.

5. Discussion

5.1. Fracture mechanism

At the macroscopic scale, all notched tensile specimens feature a slant fracture surface (e.g. see Fig. 15). Here, our attention is limited to SEM images of the fracture surfaces to shed some light on the underlying fracture mechanism. For our dis-
cussion, it is worth recalling that the main alloying elements of aluminum 6260 are magnesium (0.45–0.7%) and silicon (0.4–0.6%), while the present Al–Mg–Si alloy system is in the precipitation hardened T6 condition.

Fig. 16 shows selected SEM images of the fracture surfaces of (a) 0°/C176 butterfly shear specimen, (b) 0° notched tensile specimen, and (c) 90° notched tensile specimen. The SEM fractograph for shear loading along the rolling direction (Fig. 16a) shows a clean surface with slip traces which is characteristic for shear fracture. The fracture surfaces of the notched specimens (Fig. 16b and c) appear to be the result of intergranular fracture (large smooth areas) in combination with some void sheet fracture (areas with small dimples). Note that the presence of a precipitate free zone near the grain boundaries would facilitate intergranular ductile fracture (see the work on a 2000 series alloy by Morgeneyer et al. (2008).

Recall from our DIC and FEA analysis that the strain fields on the specimen surface indicated the formation of shear bands for all shear specimens. In the case of the notched tensile specimens, shear bands can also form in a plane perpendicular to the specimen width direction (e.g. Spencer et al., 2002b). Thus, these shear bands are not detectible through surface strain measurements. We therefore interrupted a notched tensile experiment on a 90° specimen after applying 98% of the displacement to fracture and prepared a cross-sectional cut for micrographic analysis (Fig. 15c). Even though the grain boundaries are not visible in the micrograph shown in Fig. 15c, one can detect the presence of a shear band from the pronounced offset on the lower specimen surface (which has no longer the smooth shape of a neck). Furthermore, the micrograph reveals that an approximately quadrangular channel begins to form within the neck. The formation of channels (i.e. macroscopic voids) is a well-known damage mechanism in metals for plane strain conditions. Recent examples are the experiments by Spencer et al. (2002a,b) on aluminum 5754 and Ghahremaninezhad and Ravi-Chandar (2011) on pure polycrystalline copper. Orowan (1949) explained the growth of channels through alternating slip. Similar to the mechanisms described by Spencer et al. (2002b), it is speculated here that co-operative shear is followed by inter- and intragranular void-sheet fracture.

![Fracture of notched tensile specimen](image)
The localization of plastic deformation within narrow shear bands (of the width of the grain size) is seen as the characteristic precursor to fracture in the present alloy. The study by Hu et al. (2008) on an Al–Mg alloy has shown that the presence of intermetallic particles has only a weak effect on the localization paths. Morgeneyer et al. (2008) explained the anisotropy in the fracture properties of an aluminum 2000 series alloy (Al–Cu–Mg system) through morphological anisotropy, i.e. aligned chains of intermetallic particles and pores along the rolling direction. For the present 6000 series alloy, we did not observe any notable morphological anisotropy (even at higher magnification). Furthermore, the inspection of micrographs did not reveal any evidence of second phase particles contributing to the fracture process. It is thus speculated that the observed anisotropy in the macroscopic strain to fracture is due to the effect of texture and possible grain shape anisotropy on the formation of shear bands and subsequent fracture.

The low strain hardening associated with the T6-state of the present material is expected to favor shear-type of slant fracture. Asserin-Lebert et al. (2005) demonstrated that heat treating a 6056 aluminum alloy can change the fracture mode of Kahn specimens from slanted to flat. They argue that the conditions for band localization are more easily fulfilled in the case of low strain hardening capability (which is characteristic for the T6 state). It is also worth noting that the observed small dimples on the fracture surface (traces of secondary voids) are typical for the T6 state. For instance, Simar et al. (2010) attributed the nucleation of secondary voids in a 6005 alloy in the T6 condition to the presence of...
small nanometer-sized dispersoids and the high flow stress. Similar to the mechanism described by Asserin-Lebert et al., 2005, these secondary voids are coupled with the shear localization process, as they favor the formation of intense deformation bands.

5.2. Critical comment on the proposed anisotropic fracture model

The proposed phenomenological modeling approach facilitates the transformation of isotropic to anisotropic fracture models. With only three additional parameters, the anisotropic MMC model is able to capture the anisotropy in ductility while maintaining the model’s good flexibility to fit experimental data covering a wide range of stress states. However, the model should be used with caution. In particular, the following points are noted:

(1) In analogy with linear transformation based anisotropic plasticity models (Barlat et al., 2003; Cazacu and Barlat, 2001; Karafillis and Boyce, 1993), the proposed anisotropic extension is a purely mathematical representation to describe the macroscopic anisotropic fracture behavior of metal sheets.

(2) The choice of the underlying isotropic weighting function is based on the Mohr–Coulomb failure model. This choice is supported by the observation of shear fracture. However, the original physical meaning of the model parameters is partially lost throughout the derivation of the MMC model. The linear transformation of the strain tensor amplifies the loss of the original physical meaning further.

(3) It is recommended to use a wide experimental database to identify the model parameters. Ideally, the experimental database is chosen such that it covers the range of model application. Note that due to the loss of the physical meaning of the model parameters, the predictive capabilities of this model are unknown.

The present work has focused on the development of a phenomenological fracture model; this approach is very efficient from a computational point of view as it is uncoupled from the plasticity model. However, it is noted that physics-based models for predicting anisotropic fracture could also be used (see Benzerga and Leblond, 2010). The trade-off between efficiency and physical motivation needs to be evaluated in future work. Moreover, it may also be worth developing a weighting function for use in a phenomenological model based on a void growth model.

6. Concluding remarks

The ductile fracture response of 2 mm thick extruded aluminum 6260-T6 sheets is investigated using a hybrid experimental–numerical approach. An extensive experimental program including experiments for both tension- and shear-dominant loading is carried out to cover a wide range of stress states and material orientations. Finite element simulations are performed of all fracture experiments to determine the evolution of the local stresses and strains prior to the onset of fracture. The main conclusions of this work are:

(1) The good agreement of the numerical predictions with all experimentally-measured force–displacement curves validates the extended Yld2000 yield function for general three-dimensional stress states proposed in the companion paper (Dunand et al., submitted for publication).

(2) The governing failure mode for shear-dominant loading is the localization of plastic deformation in shear bands. A particularly strong effect of the material orientation on the instant of shear band formation has been observed. This anisotropy is accurately predicted by the plasticity model.

(3) In all fracture experiments, a strong dependence of the equivalent plastic strain to fracture on the material orientation is observed. The framework of uncoupled isotropic phenomenological fracture models (e.g. Fischer et al., 1995) is extended to account for the effect of anisotropy at the macroscopic level. The underlying damage accumulation law makes use of an isotropic stress-state dependent weighting function along with a scalar anisotropic plastic strain measure.

(4) It is shown that the non-associated anisotropic fracture model is able to predict the material direction dependency of the fracture strain with good accuracy for all experiments. Specifically, we made use of the isotropic Modified Mohr–Coulomb (MMC) weighting function, while the scalar anisotropic plastic strain measure is defined through the linear transformation of the plastic strain vector before applying the von Mises equivalent plastic strain definition.

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Appendix A. Characterization of the stress state

The first invariant $I_1$ of the Cauchy stress tensor $\sigma$ and the second and third invariants ($J_2, J_3$) of the Cauchy stress deviator are defined as:

$$ I_1 = tr \sigma, \quad J_2 = \frac{1}{2} s : s, \quad J_3 = det(s), $$

where $s = \sigma - \frac{(tr \sigma)}{3} I$, denotes the deviatoric Cauchy stress tensor. The hydrostatic pressure $p$, the mean stress $\sigma_m$ and the Mises equivalent stress $\sigma_{VM}$ are defined as

$$ p = -\sigma_m = -\frac{I_1}{3} \quad \text{and} \quad \sigma_{VM} = \sqrt{3J_2}. $$

The stress triaxiality is then defined by normalizing the hydrostatic pressure by the von Mises stress,

$$ \eta = -\frac{p}{\sigma_{VM}} = \frac{\sigma_m}{\sigma_{VM}} = \frac{I_1}{3\sqrt{3J_2}}, $$

with $-\infty < \eta < \infty$. The so-called Lode angle $\theta$ is related to the normalized third invariant,

$$ \theta = \frac{1}{3} \cos^{-1} \left( \frac{3\sqrt{3}J_2}{2J_1^2} \right), \quad 0 \leq \theta \leq \frac{\pi}{3}. $$

The Lode angle $\theta$ characterizes the position of the second principal stress $\sigma_{II}$ with respect to the maximum and minimum principal stresses $\sigma_I$ and $\sigma_{III}$. Note that $\theta = \frac{\pi}{6}$ for $\sigma_I = \sigma_{II} > \sigma_{III}$ and $\theta = 0$ for $\sigma_I > \sigma_{II} = \sigma_{III}$, while $\theta = \frac{\pi}{2}$ for $\sigma_{II} = \frac{\sigma_{III} + \sigma_I}{2}$. For convenience, the dimensionless Lode angle parameter $\tilde{\theta}$ is introduced as

$$ \tilde{\theta} = 1 - \frac{6\theta}{\pi}, \quad -1 \leq \tilde{\theta} \leq 1. $$

References


