Plasticity of formable all-metal sandwich sheets: Virtual experiments and constitutive modeling

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1. Introduction

Sandwich construction enables the design of structures of an exceptionally high bending stiffness-to-weight ratio. The underlying design concept is the separation of two flat sheets by a much thicker core layer of low density. Sheet metal and fiber reinforced plastics are typically chosen as face sheet materials, while the choice of the low density core layer material is far more complex: in addition to basic elastic and weight properties of the core layer, multi-functionality (e.g., thermal, acoustic and energy absorption properties) as well as manufacturing considerations come into play (Evans et al., 1998).

To satisfy the requirement of low density (as compared to the face sheet material), lightweight bulk materials such as balsa wood (e.g., Vural and Ravichandran, 2003; Cantwell and Davies, 1996) or polymers may be used directly in combination with steel or aluminum skins (e.g., Palkowski and Lange, 2007). As an alternative to low density bulk materials, man-made porous materials find wide spread use. Hexagonal honeycombs are still the most widely used constructed sandwich core material. The elastic structure-property relationships for honeycombs are known for several decades (e.g., Kelsey et al., 1958; Gibson and Ashby, 1988) and most research on honeycombs focused on understanding and modeling their large deformation behavior (McFarland, 1963; Wierzbicki and Abramowicz, 1983; Papka and Kyriakides, 1994; Chung and Waas, 2002; Mohr and Doyoyo, 2004). Extensive research has been performed during the past two decades on the mechanical behavior of polymeric and metallic foams and their use in sandwich structures (e.g., Gibson and Ashby, 1988; Bart-Smith et al., 1998; Ashby et al., 2000; Bastawros et al., 2000; Dillard et al., 2005; Gong et al., 2005, Tan et al., 2005a,b; Demiray et al., 2007; Ridha and Shim, 2008; Luxner et al., 2009). However, more recent results suggested that lattice materials are more efficient from a mechanical point of view (e.g., Wicks and Hutchinson, 2001; Deshpande and Fleck, 2001; Evans et al., 2001; Chiras et al., 2002; Liu and Lu, 2004; Queheillalt and Ladley, 2005; Mohr, 2005; Hutchinson and Fleck, 2006; Liu et al., 2006). Zupan et al. (2003) investigated the through-thickness compression response of egg-box structures and reported a higher specific energy absorption as compared to metal foams. Tokura and Hagiwara (2010) analyzed the stiffness and strength of a stamped two-layer panel material that featured an egg-box like periodic array of domes of pyramidal shape with triangular base. They found that it is critically important to account for local thickness changes and work hardening during stamping when estimating the bending strength of stamped core layers.
Sandwich construction is mostly limited to flat panel-type of structures. Sandwich structures with curved mid-planes are difficult to make. One manual manufacturing option is to form the face sheets and core structure separately into a three-dimensional shape before bonding all layers together to form the stiff sandwich structure (Bitzer, 1997). As an alternative to this labor intensive manufacturing process, curved sandwich structures can be made from flat sandwich sheets using conventional sheet metal forming processes such as stamping and deep drawing (e.g., Mohr and Straza, 2005; Parsa et al., 2010; Seong et al., 2010; Carrado et al., 2011). In the present study, we investigate the mechanical behavior of a core structure that is composed of two bi-directionally corrugated steel layers (Fig. 1). This structure has several advantages in forming operations (Straza, 2007). In particular, it delays the dimpling failure of the compressed face sheet during bending and compression dominated stages of forming. Fig. 2 shows an illustration of the successful draw bending of a prototype made from this material.

As an alternative to physical experiments, numerical simulations of a representative unit cell of the sandwich material are performed to investigate the effective behavior of this cellular material under multi-axial loading conditions. Predicting the effective behavior of cellular materials based on the FE analysis of the underlying unit cell (for periodic media) or the representative volume element for statistically homogeneous microstructures has been successfully used by several research groups. For example, Mohr and Doyoyo (2004) investigated the crushing response of a sintered stainless steel sphere assembly (similar to a closed-cell foam) based on X-ray tomography images. It is worth mentioning that unit cell and RVE computations are only representative for the material behavior if the microstructures remain mechanically stable. In the case of instabilities (which are frequent in cellular materials of low relative density), a careful analysis of the type of instabilities is needed to check the validity of the homogenized material description (see e.g., Triantafyllidis and Schraad, 1998).

Thanks to the good accuracy of unit cell computations for constructed cellular materials with stable microstructures, it may be envisioned to use two-scale finite element models for large scale structural analysis. In other words, a unit cell model could be assigned to each integration point of the large scale model (e.g., Mohr, 2006). However, in view of computational efficiency, the feasibility of this approach appears to be limited to very simple microstructures. In other words, phenomenological macroscopic constitutive models are still needed to describe the effective behavior of cellular materials with a complex microstructure. Deshpande and Fleck (2000) developed an isotropic yield function for foams where the square of the mean stress is added to the square of the von Mises equivalent stress. They made use of an associated flow rule along with a stress-state dependent isotropic hardening law. Xue and Hutchinson (2004) proposed an anisotropic constitutive model for metallic sandwich cores by adding three square normal stress terms to the Hill’48 equivalent stress definition. Note that similar to the physics-based Gurson (1977) model for porous metals, the phenomenological Deshpande–Fleck and Xue–Hutchinson models incorporate the effect of the mean stress on yield through even terms.

Based on the assumption that the effect of in-plane stresses may be neglected in sandwich structures, Mohr and Doyoyo (2004) proposed a non-associated plasticity model to describe the large deformation response of low density honeycombs. A generalized anisotropic plasticity model for sandwich plate cores has been presented by Xue et al. (2005). They normalized all stress tensor components to define an elliptical yield function (which is an even function of the stress tensor). Due to the normalization, it is easy to introduce yield surface shape changes (distortional hardening) in addition to isotropic hardening. Xue et al. (2005) also show extensive results from unit cell simulations which support the introduction of distortional hardening. Micromechanical models of truss-lattice materials (Mohr, 2005) and hexagonal honeycombs (Mohr, 2006) explain distortional hardening at the macroscopic level through the evolution of the unit cell geometry as the material is subject to finite strains. In sheet metal plasticity, changing Lankford coefficients are seen as an indicator for texture changes (e.g.,
A general kinematic-distortional hardening modeling framework can be found in Ortiz and Popov (1983). Aretz (2008) proposed a simple isotropic-distortional hardening model, where the shape coefficients of a non-quadratic plane stress yield surface (Aretz, 2004) are expressed as a function of the equivalent plastic strain. To account for the direction dependent strain hardening with constant $r$-values, Stoughton and Yoon (2009) made use of a non-associated flow rule and integrated four stress–strain functions to control the evolution of the shape and size of a Hill’48 yield criterion.

The present work is organized as follows. In Section 2, a detailed unit cell model of the bi-directionally corrugated sandwich sheet material is constructed accounting for thickness non-uniformities and residual stresses associated with the manufacturing of the core structure. This model is then used in Section 3 to determine the macroscopic stress–strain response of the sandwich material for out-of-plane compression and shear loading and uni-axial and bi-axial in-plane loading. A macroscopic constitutive model is then proposed in Section 4 which includes an anisotropic pressure dependent yield surface along with an isotropic-distortional hardening model. In Section 5, the macroscopic model predictions are compared with the unit cell simulations and discussed in detail, before presenting the conclusions in Section 6.

### 2. Computational models for virtual experiments

The term “virtual experiments” is used to refer to the estimation of the effective properties of the proposed sandwich material based on detailed numerical simulations. Based on the results from virtual experiments, a homogeneous equivalent “macroscopic model” of the unit cell will be developed for large scale structural analysis. Note that the detailed modeling approach is only feasible with reasonable computational effort at the unit cell level, whereas a macroscopic model is required for the design of structures made from sandwich materials. The mechanical behavior of sheet materials in forming and crash simulations can be predicted with remarkably high accuracy using state-of-the-art computational models. Since the proposed sandwich material corresponds to a sheet metal assembly, it is expected that the virtual experiments will provide representative estimates of its effective behavior.

#### 2.1. Description of the sandwich material

The material coordinate system $(e_L, e_W, e_T)$ as shown in Fig. 1 is introduced to describe the microstructure. The coordinate axis $e_T$ is aligned with the thickness direction of the sandwich sheet material (out-of-plane direction) whereas $e_W$ and $e_L$ denote the so-called in-plane directions. The $L$-direction is parallel to the connecting line of two neighboring domes while the $W$-direction is defined as $e_W = e_T \times e_L$. The core structure features five symmetry planes; the bi-layer assembly is symmetric with respect to the central
the present study; for example, Cu-brazing at a temperature of 1100 °C for 1 h is expected to introduce a phase change (Fe–C eutectic temperature is 725 °C).

3 Negligence of stress relaxation and thermal residual stresses; these two complex phenomena would also need to be taken into account when modeling a real brazed material.

4 Assumption of rigid braze joints; it is assumed that the braze joints are very thin (and strong), such that the deformation in these joints is negligibly small with respect to the deformation of the core layers.

An attempt was made to confront the results from virtual experiments with experiments on real prototypes. It was found that point (2) presents a first order effect which makes it almost impossible to achieve good agreement. Note that the stress–strain response for the brazed material is expected to be different from that assumed in our models because of the possible effect of phase changes, recrystallization, stress relaxation and thermal residual stresses. Furthermore, we could only make small coupon size samples which were not suitable for biaxial testing. Here, virtual experiments will be performed for in-plane and out-of-plane loading.

2.3. Manufacturing simulations

A finite element model of a unit cell of the core structure is obtained after simulating three manufacturing steps: (1) the stamping of flat sheets to create the dimpled shaped layers, (2) the forming of flat bonding lands on each layer, and (3) elastic spring-back. The residual plastic strain fields are imported from one step to the next one. Once the core structure is created, flat face sheets are added.

Step #1: Stamping. The colored dashed rectangles in Fig. 3a indicate the size of the unit cell models which are used to perform the virtual simulations. The green lines define the smallest model; the model defined by the red rectangle is twice as wide as the green model, while the blue model is twice as wide. The different dimensions are needed to facilitate the definition of periodic boundary conditions (which depend on the specific loading case to be studied). In the case of the small green model, two punches (male die) along with their receiving female dies are needed for the stamping of this unit cell. All forming tools are modeled as analytical rigid surfaces. A mesh with five first-order solid elements (type C3D8R from the Abaqus element library) in the thickness direction is chosen to account for high through-thickness stresses as well as through-thickness necking. The receiving dies are fixed in space while the punches move along the T-direction (Fig. 5). To guarantee quasi-static conditions throughout the stamping simulations, the punch velocity increases linearly from 0 to 1 m/s over a time interval of 40 μs. Subsequently, it is kept constant until the maximum stamping depth is reached. A kinematic contact formulation with a friction coefficient of 0.1 is employed to model the contact between the tools and the sheet surfaces. Throughout stamping, the in-plane displacement component uL is set to zero for all nodes on the boundary surface of normal eL, while uw = 0 on all boundaries of normal vector eW.

Step #2: Forming of the bonding land and joining of the core layers and face sheets. After completing the previous stamping simulations (step #1, explicit time integration), an additional forming step is introduced (step #2, explicit time
integration) where flat rigid plates are used to flatten the bonding lands (Fig. 2b). A first rigid plate applies a pressure to the bottom surface of the corrugated sheet until the resulting material thickness below the centers of the punches equals about 80% of the initial sheet thickness. Similarly, a second rigid plate is used to apply a pressure to the top surface of the corrugated sheet. The simulation is stopped as the initial sheet thickness above the dies equals about 90% of the initial sheet thickness. A tie contact model (Abaqus, 2008) is used to join the core layers to each other as well as to the respective top and bottom face sheets. The flatness of the bonding lands (contact areas) is important to avoid an artificial mesh distortion when using the tie contact model. In reality, the flatness of the bonding lands is also important as it enhances braze joint strength.

Step #3: Springback analysis. After joining all layers together with the tie contact, a spring back analysis is performed (step #3, implicit time integration). The final shape and dimensional changes associated with spring back are negligibly small for the present design, but it is still important to compute a macroscopically stress-free configuration before starting any virtual experiments on this unit cell model.

2.4. Models for virtual experiments

The outcome of the manufacturing simulations is a unit cell model which includes the residual stress and plastic strain fields due to manufacturing. Here, we briefly describe the boundary conditions and output variables that have been used to characterize the effective mechanical behavior of the sandwich material. All simulation results will be presented in Section 3.

2.4.1. Out-of-plane compression

The red unit cell model is positioned between two flat rigid plates (of normal \( e_z \)). The upper plate moves along the T-axis, while the lower plate is fixed in space. To guarantee quasi-static conditions throughout the simulation (with explicit time integration), the loading velocity increases linearly from 0 to 0.003 m/s maximum over a time interval of 5 ms and is kept constant until the end of the virtual experiment. A kinematic contact formulation with a friction coefficient of 0.1 is employed to model the contact between the tools and the sheet surfaces. Periodic boundary conditions are defined for all nodes positioned on the lateral boundaries (of normal \( e_w \) and \( e_L \)).

The effective engineering out-of-plane normal stress is defined as

\[
\Sigma_{\text{eff}} = \frac{F_c}{A_t}
\]

with the initial cross-sectional area \( A_t = \frac{1}{2} D_t^2 \). The corresponding out-of-plane engineering normal strain is defined as

\[
\varepsilon_{\text{eff}} = \frac{u_t}{c_t}
\]

where \( u_t \) and \( F_c \) define the displacement and force applied by the moving upper plane.

2.4.2. Out-of-plane shear

We limit our attention to shear loading along the W- and L-directions. We expect a similar shear response for intermediate direction because of the hexagonal symmetry of the core structure. A possible direction dependency of the shear response could only arise from the small anisotropy in the basis material and possible microstructural instabilities that could result in change of deformation mechanism (unlikely for this thick-walled core structure).

The red unit cell model is used for shear in the (L,T)-plane, while the blue unit cell model is used for shear in the (W,T)-plane. The boundary conditions for shear loading in the (L,T)-plane are:

- Periodicity of the structure along the L-direction: the displacements of a node on a first (W,T)-boundary plane are identical to the displacements of the corresponding node with the same \( x_w \) and \( x_T \) coordinate on the second (W,T)-boundary plane.
- Symmetry of the mechanical problem along the W-direction: the in-plane displacement \( u_w \) of all nodes on the (L,T)-boundary planes is set to zero.
- Symmetry of the mechanical problem along the L-direction: the in-plane displacement \( u_L \) of all nodes on the (W,T)-boundary planes is set to zero.

The shear load is introduced by a displacement \( u_L \) along the L-direction applied to all nodes on the (W,T)-boundary planes of the top face or by displacement \( u_w \) along the W-direction applied to all nodes on the (L,T)-boundary planes of the top face for shearing on the other direction. To guarantee quasi-static conditions throughout the shearing simulations, the loading velocity increases linearly from 0 to 0.003 m/s maximum over a time interval of 5 ms and is kept constant until the end of the step. Denoting the corresponding reaction forces as \( F_L \) and \( F_W \), we define the out-of-plane engineering shear stresses and strains as

\[
\tau_{LT} = \frac{F_L}{\frac{1}{2} D_t^2} \quad \text{and} \quad \tau_{WT} = \frac{F_W}{\frac{1}{2} D_t^2}
\]

and

\[
\Gamma_{LT} = \frac{u_L}{c_t} \quad \text{and} \quad \Gamma_{WT} = \frac{u_W}{c_t}
\]

2.4.3. Uniaxial in-plane loading

The green model is used for in-plane loading. Due to the symmetry of the mechanical problem with respect to the L-W-plane, a green model with one core layer and one face sheet is used for in-plane simulations. The specific boundary conditions are:

- The in-plane displacement along the L-direction of all nodes on the first (W,T)-boundary plane is set to zero. A kinematic constraint is imposed on all nodes on the second (W,T)-boundary
plane to guarantee that the plane remains flat. The displacement of these nodes is denoted as $u_c$.

Analogously, the in-plane displacement along the W-direction of all nodes on the first (L,T)-boundary plane is set to zero. A kinematic constraint is imposed on all nodes on the second (L,T)-boundary plane to guarantee that the plane remains flat. The displacement of these nodes is denoted as $u_W$.

The out-of-plane displacement $u_T$ of a set of nodes located on top of the core dimples (i.e., the center of the sandwich core) is set to zero.

For uniaxial tension and compression along the L-direction, we prescribe $u_L$ while $u_W$ is free (Fig. 6a). Conversely, we prescribe $u_W$ and leave $u_L$ free for uniaxial loading along the W-direction (Fig. 6b).

In the case of in-plane loading, the engineering stresses for the face sheets, core layer and the entire sandwich material are

$$\sigma_L = \frac{F_L}{A_L}, \quad \sigma_W = \frac{F_W}{A_W} \quad \text{and} \quad \sigma_L = \frac{F_L + F_W}{A_L + A_W}$$

with the initial cross-sectional areas $A_L = \sqrt{3}Dt$, $A_W = \sqrt{3}DC$, $A_W' = \frac{D}{2}$, and $A_L' = \frac{D}{2}$. The corresponding macroscopic engineering strains read

$$E_L = \frac{u_L}{D} \quad \text{and} \quad E_W = \frac{u_W}{\sqrt{3}D}$$

2.4.4. Combined in-plane loading

The same unit cell and boundary conditions as for uniaxial in-plane loading are used to perform virtual experiments for combined in-plane loading. We introduce the biaxial loading angle to describe the ratio of in-plane strains,$\tan \beta = \frac{\partial E_W}{\partial E_L} = \sqrt{3} \frac{du_W}{du_L}$

The virtual experiments are then carried out for radial loading (i.e., monotonic loading with constant $\beta$).

3. Results from virtual experiments

The program of virtual experiments includes both out-of-plane and in-plane loading. The emphasis of the present work is on in-plane loading. Selected results for out-of-plane loading are included to shed some light on the overall mechanical behavior of the bi-directionally corrugated core material.

3.1. Uniaxial out-of-plane compression

The macroscopic response for out-of-plane loading is shown in Fig. 7. The curve starts with a linear elastic regime followed by a monotonically hardening plastic response as the stress exceeds 30 MPa. Observe from the deformation snapshots taken throughout different stages of loading that the dome height is progressively reduced. This induces a state of compression in the flat bonding land area in the center of the core structure where regions of very high plastic strains develop. It is important to note that the out-of-plane compressive response of the present material is very different from that of traditional cellular materials. We observe no peak stress (which indicates the absence of plastic collapse of the cellular microstructure); no plateau regime (which indicates the absence of progressive folding of the cellular microstructure).

However, as for traditional cellular materials, densification is expected to occur for the present material. The simulations were stopped too early to see the effect of densification on the stress-strain curve. The careful comparison of snapshots #2 and #3 reveals that a contact zone develops between the upper and lower
domes which corresponds to an increase of the apparent size of the bonding lands.

3.2. Out-of-plane shear

Virtual experiments for out-of-plane shear loading are performed in the L-T- and W-T-planes. The corresponding engineering shear stress–strain curves (Fig. 8) are almost the same (stress level is about 3% higher for L-direction). We observe an initial yield point at around 15 MPa. Thereafter, the stress continues to increase monotonically. Careful inspection of the deformed shapes shows a distortion on the dome structure due to out-of-plane shear. Observe the apparent jump in the displacement field near the center of the vertical unit cell boundaries. This is a three-dimensional effect. For example, for shear along the W-direction, the \( u_W \)-displacement field is continuous and satisfied the periodicity conditions along the boundaries, but it varies along the L-direction which gives the impression of a jump when looking at the projection on the W-T-plane. The highest strains are observed near the braze joints which is expected as the net cross-section is the smallest in that area.

3.3. Uniaxial in-plane tension

Fig. 9 shows the engineering stress–strain curves for uniaxial in-plane tension. The red curves show the results for tension along the L-direction, while the blue curves correspond to tension along the W-direction. The effective stress–strain curves are monotonically increasing and their shapes resemble that of a conventional metal. For tension in the L-direction, the initial yield stress is about 130 MPa for the entire sandwich material and reaches a value of about 160 MPa at an engineering strain of 0.15. Fig. 10a elucidates the contribution of the face sheets and the core layers to the overall axial force of the sandwich material. For tension along the L-direction, the face sheets contribute about 57% to the overall force level, while the core layers contribute the other 43%. This strong contribution of the core layer to the in-plane deformation resistance of the sandwich material is a very special feature of the bi-directionally corrugated core structure. Note that for most traditional sandwich material it is assumed that the contribution of the core layer to the in-plane stiffness and strength is negligible.

It is worth noting that the same basis material (alloy and thickness) is used for the face sheets and the core structure. This also implies that the overall weight of the sandwich material is equally split between the face sheets and the core structure. Therefore, the weight-specific response of the face sheets is more effective than that of the core structure (for uniaxial tension), but nonetheless the latter may still be seen as very high for a cellular material. In Fig. 9c, we also plotted the stress–strain response of the basis material for reference (dashed lines). The comparison of the dashed and solid curves reveals that a higher effort is needed to deform the face sheets in the sandwich material as compared to testing these independently from the core structure. The coupling with the core structure results in non-uniform deformation fields (see contour plots in Fig. 9d) which increases the plastic work required for axial straining of the face sheets.
The small differences between the two curves shown in Fig. 9a indicate some anisotropy in the sandwich material response. The breakdown into the contributions of the faces and core layers (Fig. 9b and c) demonstrates that this anisotropy may be attributed to the face sheet response. However, as shown by the dashed lines in Fig. 9c, this anisotropy is not only due to the original (texture related) anisotropy in the basis material. It is also due to the interaction with the core structure.

The core structure is compressible (from a macroscopic point of view) and hence the definition of an $r$-value is not very meaningful to describe the anisotropy. Instead, we determine an apparent plastic Poisson’s ratio from a plot of the width versus axial strain (Fig. 10b). For the present material, we obtain $v_{LW}^p = -0.33$ and $v_{EW}^p = -0.28$ for uniaxial tension along the L- and W-directions, respectively.

3.4. Uniaxial in-plane compression

The effective engineering stress–strain curves for uniaxial compression are shown in Fig. 11a. They both exhibit a maximum in stress followed by a slightly decreasing stress level. The initial small strain response is very similar to that for uniaxial tension and we observe an initial yield stress of about 130 MPa. The shal-

![Fig. 8. Out-of-plane shear: (a) macroscopic engineering shear stress–strain curves; (b) side views of deformed configurations corresponding to the points labeled in the stress–strain curves.](image)

the low peak in stress is associated with the out-of-plane deformation of the face and core sheets which may be considered as a local collapse mode of the sandwich microstructure. This deformation mode is local in the sense that the sandwich mid-plane remains flat (as imposed by the symmetry conditions). The local bending stiffness of the core sheet is determined by the dimple pattern. In the case of compressive loading along the W-direction, less effort (as compared to the L-direction) is required to initiate an out-of-plane deformation mode as a plastic hinge can easily form perpendicularly to the loading direction (which corresponds to the expected orientation of a plastic hinge). This is due to the fact that the domes are positioned such that this hinge line can form between the domes i.e., in the area where the corrugated sheets exhibit the lowest plastic bending resistance. In the case of compression along the L-direction, the dome positioning prohibits the formation of a hinge line perpendicularly to the loading direction. This explains as to why the unit cell is more distorted for compression along the L-direction as compared to the W-direction. Fig. 11 indicates that the decrease in stress level is mostly due to the folding of the core structure for W-compression. In the case of L-compression, the macroscopic deformation resistance of the core structure remains more or less constant which is consistent with the described plastic hinge mechanism.
3.5. Biaxial in-plane behavior

We limit our attention to states of loading that are of interest to sheet metal forming. Moreover, we assume that the effect of in-plane anisotropy on the mechanical properties is small and focus on bi-axial in-plane loadings of positive strain along the W-direction ($\beta = 0^\circ$/146$^\circ$) and $\beta = 180^\circ$/146$^\circ$) for the sandwich structure, the core structure and the face sheet.

In addition, in view of constructing an isotropic macroscopic yield surface, three intermediate loading angles are considered:

- $\beta = 0^\circ$ (transverse plane strain tension along L-direction)
- $\beta = 180^\circ$ (transverse plane strain compression along L-direction)
- $\beta = 45^\circ$ (equi-biaxial stretching)
- $\beta = 135^\circ$ (in-plane shear, i.e., equal L-compression and W-stretching)

In addition, in view of constructing an isotropic macroscopic yield surface, three intermediate loading angles are considered:

- $\beta = 11.3^\circ$
- $\beta = 101.3^\circ$, i.e., $\frac{dW}{d\varepsilon_T} = \frac{1}{\tan(101.3^\circ)} = -0.2$ which is close to uniaxial tension along the W-direction
- $\beta = 168.7^\circ$, $\frac{dW}{d\varepsilon_T} = -0.2$

Fig. 12a summarizes all measured engineering stress–strain curves for the different loading cases of the biaxial experiments for the sandwich structure, the core structure and the face sheet. The red dotted lines recall the results for uniaxial tension. All curves for the L-direction are in hierarchical order with respect to $\beta$ as expected for a conventional engineering material. The stress level for transverse plane strain tension ($\beta = 0^\circ$) is the highest which is a common feature of materials with a convex yield surface and an associated flow rule. Similarly, the stress level for transverse plane strain compression ($\beta = 180^\circ$) is the lowest. The stress along the L-direction is almost zero for $\beta = 101.3^\circ$ which is consistent with the observation that the corresponding stress–strain curve for the W-direction coincides with that for uniaxial tension (Fig. 12a).

The contributions of the core structure and face sheet to the overall material response are also shown in Fig. 12a. A hierarchical order of the stress–strain curves for the L-direction is observed at this level. However, we note that the W-stress–strain curves for $\beta = 45^\circ$ (tension–tension) and $\beta = 135^\circ$ (compression–tension) do intersect when considering the core structure only. The snapshots of various stages of loading in Fig. 12b reveal that the sandwich structure is distorted for $\beta = 135^\circ$ which might explain the lower apparent strain hardening of the core layer as compared to $\beta = 45^\circ$ where the core layer appears to be stretched (and flattened) in a more uniform manner. It is worth noting that the face sheets remain approximately flat (i.e., the waviness is smaller than the face sheet thickness) when both in-plane normal stresses are positive. As the second principal stress becomes negative (i.e., compression along the L-direction), we observe local out-of-plane deformation modes (see deformed configurations for $\beta > 101.3^\circ$).

3.6. Volume change of core structure

The volume change is determined from the displacement $u_T$ along the T-direction of the nodes located one the inner surface of the face sheets. At a given time step, the average surface displacement $u_T$ is determined and used to compute the engineering thickness strain. The corresponding plastic volume change is expressed through the volumetric strain.

![Graphs and diagrams showing stress-strain curves and deformation modes for different loading cases.](image-url)
Except for the initial phase of uniaxial compression and of the plastic work for all virtual in-plane experiments performed.

\[ E_p = (1 + E_v^p) (1 + E_{w}^p) (1 + E_t^p) - 1 \]  

(10)

Fig. 10 shows a plot of the plastic volumetric strain as a function of the plastic work for all virtual in-plane experiments performed. Except for the initial phase of uniaxial compression and \( \beta = 180^\circ \), we observe a volume reduction (compaction). This is expected for tensile loading conditions, but negative volumetric strains are also observed for compression-dominated loading such as \( \beta = 135^\circ \) and \( \beta = 168.7^\circ \). It is tentatively explained by the distortion of the compressed face sheet which can accommodate a local increase in core thickness, while the increase of the average core thickness is much smaller.

4. Phenomenological macroscopic constitutive model

4.1. Modeling approach

The goal is to describe the macroscopic behavior of the sandwich sheet material using a composite shell model. A composite shell model assumes that the effect of the out-of-plane normal stress is negligible which is a strong but typical assumption made in the context of thin-walled structures. Different constitutive models are assigned to the thickness integration points of the composite shell element. We therefore need to provide constitutive models that describe the effective behavior of the face sheets and the core structure when built into a sandwich structure. As an alternative, one could consider the entire sandwich sheet as a homogeneous medium and develop a single constitutive model only. However, such a description would be suitable for membrane loading only whereas it is expected to break down in the case of bending loading. The constitutive model for the face sheet basis material is known, but it provides only a poor approximation of the effective behavior of the face sheets when these are integrated into a sandwich structure.

4.2. Notation and kinematics

The constitutive equations are written in the material coordinate system which is defined through the longitudinal in-plane direction (L), the width in-plane direction (W) and the thickness direction (T). The Cauchy stress vector summarizes the non-zero stress components in that coordinate frame, \( \sigma = [\sigma_{LL}, \sigma_{WW}, \sigma_{TW}]^T \), while a standard co-rotational formulation is used to update the orientation of the material coordinate frame as the shell element is subject to large rotations and distortions (Abaqus, 2008). The work-conjugate logarithmic strain components are summarized by the strain vector, \( \varepsilon = [\varepsilon_{LL}, \varepsilon_{WW}, \varepsilon_{TW}]^T \). A superscript ‘p’ is used to denote the corresponding plastic strains, \( \varepsilon^p \). Bold lower case letters are used to denote vectors, while second-order tensors are denoted by bold letters. Square brackets are used exclusively to indicate the argument of a function, e.g., \( f = f(x) \).

4.3. Elastic constitutive equation

The core structure features hexagonal in-plane symmetry. Neglecting possible elastic anisotropy in the basis material, we can therefore use an isotropic elasticity model to describe the effective in-plane behavior, for the core structure as well as for the face sheets,

\[ \sigma = C(\varepsilon - \varepsilon^p) \]  

with

\[ C = \frac{E_m}{1 - \nu^2} \left( e_1 \otimes e_1 + e_2 \otimes e_2 + \nu e_1 \otimes e_2 + \nu e_2 \otimes e_1 + \frac{1 - \nu}{2} e_3 \otimes e_3 \right) \]  

(12)

\( E_m \) and \( \nu \) denote the elastic modulus and Poisson’s ratio for uniaxial in-plane loading, while \( e_1 = [1 0 0]^T, e_2 = [0 1 0]^T, e_3 = [0 0 1]^T \) and .

4.4. Macroscopic yield surface

The yield function will be chosen such that it defines the envelopes of equal plastic work (Table 2). We thus computed the plastic work per initial unit volume for each virtual experiment and plotted the corresponding true stress data points \( [\sigma_{WW}, \sigma_{L}] \) for selected amounts of plastic work in Fig. 14 (face sheets) and Fig. 15 (core structure). Note that we assumed plastic incompressibility for the face sheets \( (\nu = 0) \), while the volumetric strains reported in Fig. 13 are used when calculating the true effective stresses for the core structure. Colored dots in Fig. 12a show the engineering stress-strain couples corresponding to the four level of plastic work densities represented in Figs. 14 and 15.
Both the core structure and the face sheets are made from a Hill’48 material and it is hence natural to choose the Hill’48 yield function as a starting point for the construction of a yield function for the cellular material. However, the data for the core structure shows a pronounced tension/compression asymmetry which cannot be represented by the Hill’48 function. As a first approximation, the tension/compression difference in our study is attributed to a linear pressure dependency of the effective inelastic material behavior. We therefore define the yield condition as,

\[ f = \sigma - K = 0 \]  

(13)

where the equivalent stress depends both on the deviatoric and diagonal terms of the Cauchy stress tensor,

\[ \bar{\sigma} = \sigma_{\text{Hill}} + 2\sigma_m \]  

(14)

with

\[ \sigma_{\text{Hill}} = \sqrt{F(\sigma_L - \sigma_T)^2 + (\sigma_W - \sigma_T)^2 + H(\sigma_L - \sigma_T)^2 + 2L\tau_{LW}^2 + 2M\tau_{TL}^2 + 2N\tau_{LT}^2} \]  

(15)

and

\[ \sigma_m = \frac{\sigma_L + \sigma_W + \sigma_T}{3} \]  

(16)

Note that the above yield function preserves the convexity of the original Hill’48 criterion as the associated Hessian matrix is not affected by the linear pressure term. In the case of plane stress, the yield function reduces to

\[ f[\sigma] = \sqrt{\sigma_L^2 + G\sigma_W^2 + H(\sigma_L - \sigma_W)^2 + 2N\tau_{LW}^2 + \frac{2}{3}(\sigma_L - \sigma_W) - k = 0} \]  

(17)

We fitted the above expression to our virtual experimental data. The solid envelopes in Figs. 14a, d and 15a, d show the fit to the data using the parameters listed in Table 3.

4.5. Distortional-isotropic hardening

Figs. 14d and 15d shows the yield envelopes for two distinct plastic work densities in a single plot. The comparison of the calibrated yield envelopes demonstrates that the elastic domain is not increasing in a self-similar manner. Instead, the shape of the yield surfaces changes substantially (distortional hardening). In Eq. (14), changes of \( k \) represent isotropic hardening, while changes in the coefficients \( F, G, H, N \) and \( \alpha \) would represent distortional hardening.

As an alternative to modeling the evolution of the yield surface coefficients (e.g., Aretz (2005)), we describe the yield surface evolution through a linear combination of two distinct yield functions \( f_1[\sigma] \) and \( f_2[\sigma] \).

\[ f[\sigma] = (1 - \delta)f_1[\sigma] + \delta f_2[\sigma] \]  

(18)

where the weighting factor \( \delta = \delta[W_{pl}] \) is defined as a function of the plastic work density. Denoting the plastic work density associated with the yield functions \( f_1 \) and \( f_2 \) as \( W_{pl}^1 \) and \( W_{pl}^2 \), respectively, we impose the order \( W_{pl}^2 > W_{pl}^1 \).
Note that the linear combination (with positive weights) of two convex functions is still convex. Furthermore, the corresponding weighted equivalent stress,

\[
\delta = \left(1 - \delta\right)\sigma_1 + \delta\sigma_2
\]

is still a homogeneous function of degree one. For plastic work densities smaller than \(W_{pl1}\) and greater than \(W_{pl2}\), we assume isotropic hardening only. The yield function evolution law may thus be written as

\[
f[\sigma] = \begin{cases} 
\sigma_1 - (1 + \delta)k_1 & \text{for } W_{pl1} \leq W_{pl} \\
(1 - \delta)(\sigma_1 - k_1) + \delta(\sigma_2 - k_2) & \text{for } W_{pl1} < W_{pl} < W_{pl2} \\
\sigma_2 - \delta k_2 & \text{for } W_{pl2} \leq W_{pl}
\end{cases}
\]

where the weighting function \(\delta > -1\) defines a monotonically increasing function of the plastic work density which fulfills the constraints \(\delta[W_{pl1}] = 0\) and \(\delta[W_{pl2}] = 1\).
The boundary values $W_{1}^{pl}$ and $W_{2}^{pl}$ are chosen such that the model provides the best overall approximation of the stress–strain response. The distortional hardening is attributed to changes in the geometry of the core structure. Geometrical changes are negligible for small macroscopic strains and hence the assumption of isotropic hardening is made in that range. The value $W_{1}^{pl}$ represents thus the work density at which the changes in the core structure geometry are no longer negligible as far as their effect on the shape of the macroscopic yield surface is concerned. The upper value $W_{2}^{pl}$ indicates another mechanism change. For $W_{pl} > W_{2}^{pl}$, necking at the microstructural level becomes important and a macroscopic description of the material response may no longer be adequate. Here, the assumption of isotropic hardening for $W_{pl} > W_{2}^{pl}$ is just made for computational robustness.

4.6. Flow rule and volume change

An associated plastic flow rule is adopted to describe the evolution of the plastic in-plane strains. Formally, we write

$$d\varepsilon_p = \frac{d\sigma}{E}$$

with the plastic multiplier $d\varepsilon_p \geq 0$. The increment in plastic work density (per initial volume) can be written as

![Fig. 13. Plastic volume change during in-plane loading for all virtual experiments performed as a function of the plastic work per initial volume. The red dashed line shows the model approximation according to Eq. (24). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image)
\[
W_{pi} = \frac{1 + \epsilon_{v}}{r/C1} (\ldots)
\]

where a constitutive equation needs to be specified to determine the evolution of the plastic (engineering) volumetric strain \(\epsilon_{v}\). Plastic incompressibility is assumed as a first approximation for the face sheets, i.e., \(\epsilon_{v} = 0\). The core layer on the other hand is considered as compressible. The change in volumetric strain is directly defined as a function of the plastic work density,

\[
d\epsilon_{v} = h[W_{pl}](dW_{pl})
\]

with \(h = 0.0008\ \text{MPa}^{-1}\) provides a reasonable approximation of the present experimental data for the core structure (see red dashed curve in Fig. 13). Note that the above expression is only valid up to the theoretical densification strain of \(\epsilon_{v} = p^* - 1\).

### 4.7. Summary of material model parameters

The proposed material model is specified through the following parameters:

- The elastic parameters, Young’s modulus \(E_m\) and Poisson’s ratio \(v\), that describe the planar isotropic elastic behavior.

**Table 2**

<table>
<thead>
<tr>
<th>Plastic strain</th>
<th>0</th>
<th>0.02</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.60</th>
<th>0.80</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>True stress (MPa)</td>
<td>310</td>
<td>352</td>
<td>388</td>
<td>421</td>
<td>465</td>
<td>489</td>
<td>508</td>
<td>535</td>
<td>555</td>
<td>571</td>
</tr>
</tbody>
</table>

![Fig. 14. Envelopes of equal plastic work (per unit initial volume) for the face sheets in the true stress plane (\(\sigma_{WW}, \sigma_{WW}\)). The open dots present the results from virtual experiments, the black solid lines in (a) and (d) represent the least square fit of the yield function given by Eq. (17). The solid envelopes in (b) and (c) have been computed based on the isotropic-distortional hardening model given by Eq. (25).](image-url)
The yield functions \( f_1 \) and \( f_2 \); each function \( f_i \) is specified through five parameters: \( G_i, H_i, N_i, a_i \) and \( K_i \).

The weighting function \( d[W_{pl}] \) which describes combined isotropic-distortional hardening.

The weighting function may be presented as a parametric or non-parametric function. For the present sandwich material, the parametric function

\[
\delta_i[W_{pl}] = \begin{cases} 
\log(a_i) \frac{W_{pl}}{W^{2}_i} + \delta_0 & \text{for } W_{pl} < W^{2}_i \\
\delta_1 \frac{W_{pl}}{W_i} + \delta_2 & \text{for } p l^2 \leq W_{pl} 
\end{cases}
\]  

provides a good approximation of the face sheet response. The yield envelopes for \( W_{pl} = 15 \text{ N/mm}^2 \) and \( W_{pl} = 30 \text{ N/mm}^2 \) shown in Fig. 14 have been computed using the above expression for the weighting function.

For the core structure, we propose the function

\[
\delta_i[W_{pl}] = \begin{cases} 
\delta_1 \frac{W_{pl}}{W^2_i} + \delta_0 & \text{for } W_{pl} < W^{2}_i \\
\delta_1 \frac{W_{pl}}{W_i} + \delta_2 & \text{for } W^{2}_i \leq W_{pl} 
\end{cases}
\]  

Table 3
Yield function parameters.

<table>
<thead>
<tr>
<th>( W_{pl} ) (N/mm(^2))</th>
<th>( G_1 ) (–)</th>
<th>( H_1 ) (–)</th>
<th>( N_1 ) (–)</th>
<th>( a_1 ) (–)</th>
<th>( k_1 ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>1</td>
<td>1.0</td>
<td>0.6</td>
<td>1.5</td>
<td>0.03</td>
</tr>
<tr>
<td>Face</td>
<td>0.5</td>
<td>0.77</td>
<td>0.53</td>
<td>1.5</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( W_{pl} ) (N/mm(^2))</th>
<th>( G_2 ) (–)</th>
<th>( H_2 ) (–)</th>
<th>( N_2 ) (–)</th>
<th>( a_2 ) (–)</th>
<th>( k_2 ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>12</td>
<td>1.2</td>
<td>0.22</td>
<td>1.5</td>
<td>-0.43</td>
</tr>
<tr>
<td>Face</td>
<td>50</td>
<td>0.85</td>
<td>0.3</td>
<td>1.5</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

Table 4
Weighting function parameters.

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( \delta_0 )</th>
<th>( \delta_1 )</th>
<th>( \delta_2 )</th>
<th>( \delta_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>-0.09</td>
<td>1.09</td>
<td>0.74</td>
<td>0.26</td>
</tr>
<tr>
<td>Face</td>
<td>0.91</td>
<td>9.09</td>
<td>0.91</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Fig. 15. Envelopes of equal plastic work (per unit initial volume) for the core structure in the true stress plane \((\sigma_W, a_i)\). The open dots present the results from virtual experiments, the black solid lines in (a) and (d) represent the least square fit of the yield function given by Eq. (17). The solid envelopes in (b) and (c) have been computed based on the isotropic-distortional hardening model given by Eq. (26).
to describe the apparent strain hardening. Fig. 15 includes the intermediate yield envelopes for $W_{pl} = 4 \text{ N/mm}^2$ and $W_{pl} = 8 \text{ N/mm}^2$ that have been determined using Eq. (18) in combination with Eq. (26). All model parameters as calibrated for the present face sheet and core materials are summarized in Tables 3 and 4.

5. Validation and discussion

The constitutive model is implemented into the finite element software Abaqus/explicit through its VUMAT user material subroutine interface. In the VUMAT code, we adopt a standard return
mapping algorithm with a backward-Euler time integration scheme (Simo and Hughes, 1998). It is subsequently used in conjunction with a composite shell element. The cross-section of the composite shell element is composed of three layers representing the top and bottom face sheets (each 0.2 mm thick) along with a 1.2 mm thick core layer. Three thickness integration points (for numerical integration with the Simpson rule) are employed per layer.

5.1. Comparison: macroscopic model versus virtual experiments

All in-plane experiments are simulated using the composite shell model. The results are reported in terms of the section normal forces $F_1$ and $F_2$ as a function of the corresponding engineering normal strains $E_1$ and $E_2$. The solid blue lines in Fig. 16 depict the results for the composite shell element while the dashed blue lines show the corresponding results from the virtual experiments. In addition, we also computed the individual contributions of the face sheets (red curves) and the core structure (black curves).

We observe good overall agreement of the force-strain curves for most loading cases. The best agreement of model and virtual experiments is observed for transverse plane strain loading ($\beta = 0^\circ$ and $\beta = 180^\circ$). For uniaxial tension and compression, the predicted force agrees well with that of the virtual experiment, but it is underestimated by up to 15% at large strains. The model predictions are less accurate for combined loading. However, the force level predictions are still reasonable when quantifying the error in absolute terms. For example, the relative error in $F_2$ exceeds 100% for $\beta = 101.3^\circ$, but the absolute difference is less than 20 N/mm. The comparison of the force-strain curves for the face sheets demonstrates a good agreement for almost all experiments. The observed differences in force level for the sandwich may thus be attributed to deficiencies in the model predictions of the effective behavior of the core structure. The plots of the yield envelopes in Fig. 15 demonstrate that the error in the core model predictions are not due to the yield functions. Instead, it is speculated that the flow rule is not very accurate. Note that all biaxial experiments are strain-driven and the flow rule therefore determines the loading path in stress space.

5.2. Discussion

An attempt was made to come up with a simple micromechanics-based two-scale finite element model of the core structure. For example, a simplified three-dimensional shell element model of the unit cell could be assigned to each thickness integration point of macroscopic composite shell model (see for instance Mohr (2006)). However, our preliminary results have shown that a three-dimensional shell element model (at the micro-scale) provides only a poor quantitative prediction of the effective stress–strain response obtained from our virtual experiments (that make use of fine solid elements). Similarly, analytical solutions of strongly simplified mechanical models of the core structure (e.g., a truncated cone of uniform thickness) turned out to be inadequate from both a qualitative and quantitative point of view. Here, we proposed a simple phenomenological modeling framework to describe the effective behavior of the face sheets and core layers, respectively. Such models are only of little value as far as their predictions are strain-driven and the flow rule therefore determines the loading path in stress space.

6. Conclusions

The mechanical behavior of a newly-developed all-metal sandwich sheet material composed of two flat face sheets and two bi-directionally corrugated core layers is investigated. The mass per unit area of this sandwich material is equally split between the core structure and the face sheets. Virtual experiments are performed using a detailed finite element model of the unit cell of the periodic material microstructure. The model accounts for the thickness variations and residual stresses due to the manufacturing of the core layers. Various combinations of in-plane loading have been applied to the unit cell, including uniaxial tension, equi-biaxial tension, in-plane shear and uniaxial compression. It is found that the core structure is used very efficiently, contributing up to 43% of the effective yield strength of the sandwich sheet material for in-plane loading.

The experimental data is used to determine the macroscopic yield surfaces based on an equal plastic work definition for both the core structure and the face sheets. An anisotropic yield surface with linear pressure dependency is proposed to approximate the experimental data. Furthermore, a new isotropic-distortional hardening modeling framework is proposed to provide a first approximation of the stress–strain response for radial loading paths. The constitutive model is implemented into a commercial finite element software and used in conjunction with a composite shell element model to describe the effective in-plane behavior of the sandwich sheet material.

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References


