Probabilistic fracture of Ti–6Al–4V made through additive layer manufacturing

Thomas Tancogne-Dejean a, b, Christian C. Roth b, c, Udisien Woy d, Dirk Mohr a, b, c, *

a Solid Mechanics Laboratory (CNRS-UMR 7649), Department of Mechanics, École Polytechnique, Palaiseau, France
b Impact and Crashworthiness Laboratory, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA
c ETH Zurich, Department of Mechanical and Process Engineering, Switzerland
d Advanced Manufacturing Research Centre, University of Sheffield, UK

A R T I C L E   I N F O

Article history:
Received 14 December 2014
Received in revised form 18 September 2015
Available online 30 September 2015

Keywords:
A. Fracture
B. Anisotropic material
C. Probability and statistics
Hosford-Coulomb

A B S T R A C T

The large deformation response of Ti–6Al–4V parts made through additive layer manufacturing is investigated. A wire-feed process is chosen instead of a powder process in an attempt to reduce the oxide contaminations of the final part. The experimental program includes uniaxial tension experiments along different part directions and fracture experiments on flat specimens with cut-outs covering stress states ranging from pure shear to equi-biaxial tension. More than 100 experiments are performed in total to characterize the randomness in the material’s fracture response. It is found that the stress-strain response of the ALM material is comparable to that of Ti–6Al–4V sheet stock, while its average ductility is substantially lower. For example, for pure shear loading, the average strain to fracture for the ALM material is 0.47, while the mill product of the same alloy failed at a strain of 0.65. A probabilistic extension of the stress state dependent Hosford–Coulomb fracture initiation model is proposed to account for the significant standard deviation in the identified strains to fracture. Microscopic and surface strain field analysis demonstrate that the initiation and propagation of ductile fracture in the ALM material is strongly affected by the presence of prior-beta grains.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

The Ti–6Al–4V alloy is the most widely used titanium alloy with applications in jet engines, airframes and biomedical implants. Consequently, its mechanical behavior has been studied extensively. For example, Zhang et al. (2007) made use of a crystal plasticity model to describe the mechanical response of a Ti–6Al–4V alloy to cyclic loading. Przybyla and McDowell (2011) introduced a microstructure-sensitive extreme value probabilistic framework to compare the fatigue failure of four different Ti–6Al–4V microstructures. Khan et al. (2012) formulated an anisotropic criterion with tension-compression asymmetry to describe the yield behavior of Ti–6Al–4V. The tension/compression asymmetry, anisotropic yielding and anisotropic strain-hardening in Ti–6Al–4V ingots has also been characterized experimentally and modeled at the

* Corresponding author. Solid Mechanics Laboratory (CNRS-UMR 7649), Department of Mechanics, École Polytechnique, Palaiseau, France.
E-mail address: mohr@mit.edu (D. Mohr).

http://dx.doi.org/10.1016/j.ijplas.2015.09.007
0749-6419/© 2015 Elsevier Ltd. All rights reserved.
macroscopic level by Tuninetti et al. (2015). A theoretical model predicting the spacing of periodic adiabatic shear bands during high speed machining of Ti–6Al–4V has been prosed by Ye et al. (2013), Li et al. (2014) observed an increase in the ductility of Ti–6Al–4V during ring expansion experiments at strain rates above about $7 \times 10^3/\text{s}$.

In aerospace engineering, Ti–6Al–4V components are traditionally manufactured through intense milling of bulk parts, the hot-forming of sheets and assembly welding (Tersing et al., 2012). Additive Layer Manufacturing (ALM) provides a promising cost-effective alternative to traditional machining. Historically, ALM has been intensively used for rapid prototyping, where shape is more important than the mechanical properties of the manufactured parts. Examples are the selective laser sintering with metal powders (Agarwala et al., 1995; Kruth et al., 2003; Levy et al., 2003) or the development of 3D printing with polymers (Levy et al., 2003; Wendel et al., 2008). The mechanical properties of components made from metal powders are often affected by contaminations that are associated with the high surface-to-volume ratio of powders. To the best of the authors’ knowledge, ALM components built from powder stock are not yet used in the safety critical load carrying structures of modern jet engines.

Wire-feed processes feature a lower surface-to-volume ratio and thus a lower risk of contamination (Brandl et al., 2008). Other advantages of wire over powder include material availability, cost and quality. Brandl et al. (2008, 2009) presented an argon flooded open ALM system composed of a Nd:YAG laser beam and a wire-feeder mounted on a 6-axis robot. Baufeld et al. (2009a, 2009b, 2010) proposed the so-called Shaped Metal Deposition (SMD) process composed of a tungsten inert gas welding torch mounted on a 6-axis robot. The ALM part is built from wire stock on a 2-axis table inside a closed chamber with argon atmosphere. As compared to laser made parts, the SMD parts feature a lower nitrogen contamination (Baufeld et al., 2011).

The basic mechanical performance of ALM materials are typically characterized through uniaxial tension experiments (e.g. Baufeld et al., 2011). In view of using ALM parts in load carrying structures, the multi-axial material response needs to be known. A first objective of the present paper is to characterize experimentally and model numerically the average large deformation response of SMD made Ti–6Al–4V. Given the stochastic nature of the fracture response of ALM materials, a second objective of this work is to formulate and calibrate a probabilistic stress-state dependent fracture initiation model for SMD made Ti–6Al–4V. In Section 2, the macro- and microstructure of an SMD produced Ti–6Al–4V box structure is characterized. Subsequently, a comprehensive plasticity and fracture testing program is executed which includes the tensile testing of smooth, notched and central hole specimens as well as selected shear, bending and punch experiments. In Section 3, a non-associated plasticity model is presented along with a probabilistic formulation of the Hosford–Coulomb fracture initiation model. Finite element simulations are performed for all experiments to determine the loading paths to fracture in terms of the stress triaxiality, the Lode parameter and the equivalent plastic strain. Based on the hybrid experimental-numerical results, the four material parameters of the probabilistic fracture initiation model are identified. The final discussion is primarily concerned with the third objective of this work, which is the comparison of the observed ALM material fracture response with that of conventional Ti–6Al–4V sheet stock, and the identification of the physical origin of the observed randomness in the ALM-made Ti–6Al–4V fracture response.

2. Experiments

2.1. ALM component

SMD is a Rolls-Royce patented technology being developed for high-value industrial applications at the University of Sheffield Advanced Manufacturing Research Centre. Fig. 1 shows a schematic and a photograph of the manufacturing process. The SMD equipment consists of a 6-axis KUKA KR16 robot with a Gas Tungsten Inert Gas (GTAW) welding head, linked to a 2-axis manipulator, housed in a full sealed chamber. A 1.2 mm diameter Ti–6Al–4V wire (part ① in Fig. 1b) is fed through the GTAW welding head (part ②) at a constant feed rate. The alloy is deposited layer by layer onto a substrate plate (part ③) of the same alloy, clamped onto the 2-axis manipulator. These have been integrated with real-time process monitoring equipment including weld vision cameras and monitors. Before welding begins, the chamber is flooded with inert argon gas (purity 99.9999%, O₂ < 20 ppm). Simulation software is used for the off-line programming of the tool path based on accurate part geometries, process parameters and the cell layout. The material is then continuously deposited at an average wire feed speed of ~1700 mm/min along the programmed tool path (blue line ④ in Fig. 1a) of the rectangular shape. The rectangle is built in layers with a tool offset of 1 mm in the vertical direction and a welding current of 170 A. The 120 mm high rectangular box structure (Fig. 2a) was completed in 15 h which corresponds to an average deposition rate of 0.45 kg/h.

2.2. Macro- and microstructure

Depending on the temperature and the exact alloy composition, titanium alloys feature hexagonal close-packed HCP (α-phase) and body-centered cubic BCC (β-phase) crystal structures. For the SMD material, the grain structure forms throughout the solidification of newly deposited molten Ti–6Al–4V wire. At high temperatures (above the alpha-to-beta transition temperature of 1000 °C for pure Ti), beta-grains are expected to grow along the highest temperature gradient. Below the alpha-to-beta transition temperature, lamellar shaped α-grains grow from the inside of the β-grains (Gil et al., 2001). Note that in the Ti–6Al–4V, the alloying elements aluminum and vanadium act as alpha- and beta-stabilizers, respectively. As a result, Ti–6Al–4V exhibits a mixed alpha-beta grain structure at room temperature.
To visualize the grain structure of the ALM made parts, small samples are extracted and polished using standard mechanical polishing procedure (abrasive paper and diamond paste) followed by silica colloidal polishing. The samples are etched using a solution of 5% of hydrofluoric acid, 40% of nitric acid and 55% of water. Fig. 2b shows a low magnification photograph of the polished and etched surface. Its horizontal and vertical axes are aligned with the material deposition direction (s-coordinate) and the layer build-up direction (z-coordinate). The dark horizontal bands derive from the $\beta/\alpha + \beta$ transus lines during subsequent layer deposition (Baufeld and Van der Biest, 2009). Their average spacing is about 1 mm which corresponds to the thickness of a single layer. In addition, the boundaries of prior beta-grains are visible at the macroscale (as highlighted through white lines in Fig. 2b). The so-called "prior $\beta$-grains" exhibit an elongated columnar shape with their major axis inclined towards the deposition direction at an angle of up to 45° with respect to the build-up direction. The columnar prior $\beta$-grains are several millimeters wide and can even attain a length of several centimeters. It is worth noting that the prior $\beta$-grains traverse through multiple material layers.

At the microscale (see SEM micrographs in Fig. 2c and d), the grain structure consists of $\alpha$-needles (dark gray) inside a $\beta$-phase (white) matrix. The needles are about 10 $\mu$m long and feature a length-to-width aspect ratio of about 10:1. Within the bulk of a prior $\beta$-grain, the $\alpha$-grains are arranged in a basket weave type of pattern (Fig. 2d). Near the prior $\beta$-grain boundaries,
The $\alpha$-needles are all parallel to each other (appear as lamellae structure), with a discontinuity in their orientation defining the prior $\beta$-grain boundaries.

The organization of the $\alpha$-needles in a basket weave structure implies morphological anisotropy of individual prior beta grains. EBSD analysis for individual prior-beta grains reveals strong basal texture (Fig. 3). The comparison of the pole figures from neighboring grains shows a significant disorientation of the anisotropy axes.

2.3. Experimental procedures

The experimental program includes the tensile testing of flat specimens with different cut-outs, V-bending, shear and punch experiments. The dimensions of all specimens used are provided in Fig. 4. The exact shape of the central hole and shear specimens have been chosen according to the material specific geometry optimization procedure detailed in Roth and Mohr (2015). In a first wire EDM machining step, multiple uniform 1.5 mm thick walled sections are machined from the four sides of the rectangular ALM box structure (Fig. 2a). In a second step, the 1.5 mm thick specimens are extracted from the flat coupons. Subsequently, the central hole in the CH-specimens is introduced through conventional CNC milling. Due to the possible effect of material orientation, the orientation of the specimen axis (shown as dashed line in Fig. 4) with respect to the build-up direction ($z$-axis) is reported.
2.3.1. Tension and shear experiments

All experiments are performed on a 250 kN MTS hydraulic testing machine. Custom-made grips are used to apply the clamping pressure onto the 10 mm long shoulder region of the specimens (gray shaded areas in Fig. 4). A random speckle pattern is applied to the specimen surfaces to measure the surface displacements through planar digital image correlation (DIC) with the software VIC2D (Correlated Solutions). The images are acquired at a frequency of 1 Hz using an AVT Pike F–505B camera with a Tamron 90 mm 1:1 macro lens. The Uniaxial Tension (UT) specimens are tested at a constant cross-head speed of 0.5 mm/min. All other tensile experiments (Notched Tension (NT) and Central Hole (CH) specimens) are loaded at 0.25 mm/min.

An even lower cross-head speed of 0.05 mm/min is used for the shear (SH) specimen to achieve a comparable equivalent plastic strain rate ($\dot{\varepsilon}_p \approx 10^{-2} \text{s}^{-1}$) in all experiments. In the case of the SH experiments, two cameras are used to observe the surface displacement fields at two different magnifications: the field of view of a first camera includes the whole specimen to

![Fig. 3. Pole figures computed from EDSD analysis of 1 × 1 mm areas within neighboring macro-grains in the s–z-plane showing the distribution of the basal plane orientation.](image-url)
compute the overall displacement with a 25 mm-long extensometer (distance between blue dots on Fig. 4d); the second camera is positioned closer to the specimen to measure the strain in the shear gage sections.

2.3.2. V-bending experiments

V-bending experiments are performed to characterize the plane strain fracture response. Fig. 5a shows a newly developed V-bending device by Roth and Mohr (2015). It features a support point spacing of 3.8 mm and makes use of a sharp central punch with a tip radius of less than 0.4 mm. The particular feature of the loading device is that the punch (part ①) remains stationary while the outer support rollers (parts ②) move downwards. This set-up has the advantage (over conventional moving punch set-ups) that the surface strains can be accurately measured through stereo digital image correlation. For this, two digital cameras with 90 mm macro lenses are positioned at an angle of about 18° with respect to each other at a distance of about 800 mm above the specimen surface. Images are acquired at a frequency of 1 Hz at a cross-head loading speed of 2.0 mm/min.

The digital image correlation software VIC3D is used to compute the surface strains in the area above the punch. The random speckle pattern featured an average speckle size of 8 pixels. After verifying that plane strain conditions prevail on the specimen surface (|εz| << εr), the logarithmic axial strain εf at the instant of crack formation is reported as the main

![Fig. 4. Drawings of 1.5 mm thick flat specimens: (a) uniaxial tension (UT), (b) notched tension (NT), (c) tension with a central hole (CH), (d) double shear (SH), (e) V-bending, and (f) punch testing. The blue and red dots show the positions of the virtual extensometers used for relative displacement and local strain measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)](image-url)
experimental result. The onset of fracture is identified either through a 30N drop in force or the appearance of macroscopic surface cracks (visible by eye).

2.3.3. Punch experiment

The miniature punch testing device shown in Fig. 5c is used to subject a 60 mm diameter disc specimen (Fig. 4f) to equibiaxial tension. The specimen is positioned onto a female die with a 25.4 mm inner diameter and held in place through a clamping ring (part ①) with eight M6 screws. In order to reduce friction during the punch test six 0.08 mm thick Teflon sheets are put between the punch (part ②) and the specimen. As for the V-bending experiment, the punch is kept stationary while the die moves downwards at a speed of 2.0 mm/min. Stereo digital image correlation is used to determine the surface strains. Fracture is detected through an abrupt drop in force.

2.4. Experimental results

2.4.1. Results for uniaxial tension

The results of the uniaxial tension experiments are evaluated up to the point where the force reaches its maximum. Using standard equations, the true stress along with the logarithmic axial and width strains are determined from each experiment. Note that the axial and width strains are determined using virtual extensometers of 8 mm length (see also blue dots on specimen drawings in Fig. 2a). Fig. 6 show a summary of all measured true stress-strain curves. The corresponding true width
versus true axial strain curves are shown on the secondary axis. Different colors in Fig. 6a indicate different specimen orientations. Due to necking prior to fracture, all curves are truncated at the force maximum.

Table 1 lists the true stresses and strains at force maximum. The experimental results show a large spread with differences in stress of \( \pm 8\% \). The differences in stress appear to be only weakly correlated with the specimen orientation, e.g. both the minimum and maximum stress at force maximum are observed for 90° specimens. The slopes of the width versus axial strain curves also vary substantially from specimen to specimen. However, there appears to be some correlation with respect to the specimen orientation. Assuming an elastic modulus of 115 MPa, a Poisson's ratio of \( v = 0.34 \), and plastic incompressibility, the corresponding Lankford ratios are calculated from the logarithmic plastic width to thickness strain ratios.

Table 1
Results from uniaxial tension (UT) experiments. The second column is the material orientation (angle between tensile and \( z \)-axis), the third and fourth columns provide the logarithmic strain and true stress at force maximum, the fifth column provides the Lankford ratios.

<table>
<thead>
<tr>
<th>Name</th>
<th>( \alpha ) [°]</th>
<th>( \varepsilon_{\text{max}} ) [-]</th>
<th>( \sigma_{\text{max}} ) [MPa]</th>
<th>( r_a ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>UT0_1</td>
<td>0</td>
<td>0.088</td>
<td>1072</td>
<td>0.46</td>
</tr>
<tr>
<td>UT0_2</td>
<td>0</td>
<td>0.069</td>
<td>1002</td>
<td>0.56</td>
</tr>
<tr>
<td>UT0_3</td>
<td>0</td>
<td>0.094</td>
<td>1058</td>
<td>0.54</td>
</tr>
<tr>
<td>UT0_4</td>
<td>0</td>
<td>0.093</td>
<td>1074</td>
<td>0.81</td>
</tr>
<tr>
<td>UT0_5</td>
<td>0</td>
<td>0.110</td>
<td>1059</td>
<td>0.55</td>
</tr>
<tr>
<td>Average:</td>
<td></td>
<td>0.09</td>
<td>1053</td>
<td>0.59</td>
</tr>
<tr>
<td>UT45_1</td>
<td>45</td>
<td>0.092</td>
<td>1007</td>
<td>1.81</td>
</tr>
<tr>
<td>UT45_2</td>
<td>45</td>
<td>0.089</td>
<td>1005</td>
<td>1.06</td>
</tr>
<tr>
<td>UT45_3</td>
<td>45</td>
<td>0.136</td>
<td>1041</td>
<td>1.11</td>
</tr>
<tr>
<td>UT45_4</td>
<td>45</td>
<td>0.086</td>
<td>1001</td>
<td>1.03</td>
</tr>
<tr>
<td>Average:</td>
<td></td>
<td>0.10</td>
<td>1014</td>
<td>1.25</td>
</tr>
<tr>
<td>UT90_1</td>
<td>90</td>
<td>0.071</td>
<td>1011</td>
<td>0.37</td>
</tr>
<tr>
<td>UT90_2</td>
<td>90</td>
<td>0.030</td>
<td>929</td>
<td>0.41</td>
</tr>
<tr>
<td>UT90_3</td>
<td>90</td>
<td>0.077</td>
<td>1087</td>
<td>0.69</td>
</tr>
<tr>
<td>UT90_4</td>
<td>90</td>
<td>0.032</td>
<td>1052</td>
<td>0.56</td>
</tr>
<tr>
<td>Average:</td>
<td></td>
<td>0.05</td>
<td>1022</td>
<td>0.51</td>
</tr>
</tbody>
</table>
\[
R = \frac{d\varepsilon_{P}}{d\varepsilon_{th}} = -\frac{\varepsilon_{P}}{(\varepsilon_{S} + \varepsilon_{P})}.
\]

The results are shown in Table 1 with the average Lankford ratios \(<r_0> = 0.59\), \(<r_{45}> = 1.25\) and \(<r_{90}> = 0.51\).

2.4.2. Results from fracture experiments

The measured force-displacement curves for notched tension (NT), tension with a central hole (CH) and shear (SH) experiments are presented in Fig. 7a and b. Recall that the reported displacement corresponds to the relative displacement of two points on the specimen shoulders (shown as blue dots on the specimen drawings in Fig. 4). The curves are truncated at the onset of fracture as characterized by a drop of the force and a visible crack on the specimen surface. As for the UT-specimens, significant randomness is observed in the force-displacement curves with standard deviations in force level of about \(\pm 6.0\%\) (NT) and \(\pm 7.2\%\) (CH), and in the displacement to fracture of 22.1\% (NT) and 19.1\% (CH). The 90\° specimens seem to provide the highest deformation resistance (for UT, NT and CH), i.e. the blue curves seem to lie slightly above those for the 0\° and 45\° specimens. No significant correlation is observed between the displacement to fracture and the specimen orientation (see also Table 2). Fig. 7c summarizes the force-displacement curves for the shear experiments. The variations in the force level are comparable to those of the NT and CH experiments, whereas the standard deviation is only 10.6\% for the displacement to fracture. In the SH experiments, fracture always occurred in one gage section first, i.e. the two shear gage sections did not fracture simultaneously (unlike for dual phase steels, see Roth and Mohr, 2015).

The DIC-measured maximum principal strains to fracture in V-bending (VB) experiments are shown in Table 3. For the five experiments performed, we observe an average principal strain to fracture of 0.19 and a standard deviation of 12.3\%. It is worth noting that the same standard deviation is observed in the measured displacement to fracture. Only one punch experiment is performed because of the limited availability of ALM material. In that experiment, a maximum principal strain to fracture of 0.17 is determined through stereo DIC.

In addition to the overall specimen responses (force--displacement curves) and the local strains to fracture, we also measured the strain fields in all experiments which will be discussed in detail in Section 6.

3. Constitutive modeling

3.1. Plasticity

A simple quadratic plasticity model is employed to provide a first approximation of the inelastic material response. Note that the effect of the third stress invariant on the plastic behavior of the mixed HCP–BCC microstructure of the Ti–6Al–4V alloy is neglected in the context of the present paper. The reader is referred to the literature (e.g. Cazacu et al., 2006) for a proper treatise of this aspect. Due to the uncertainty in the ALM material response, it is still considered as a second order effect in the context of the present work.

The constitutive equations are presented in the orthogonal material coordinate system \(\{\mathbf{e}_z, \mathbf{e}_x, \mathbf{e}_y\}\), with the orthogonal unit vectors \(\mathbf{e}_z, \mathbf{e}_x\) and \(\mathbf{e}_y\) being aligned with the deposition, build-up and thickness directions, respectively. Vector notation is employed to denote the Cauchy stress tensor and the logarithmic strain tensor components, i.e.

\[
\sigma = [\sigma_{zz} \sigma_{zt} \sigma_{zt} \sigma_{zt} \sigma_{zt} \sigma_{zt}]^T
\]

and

\[
\varepsilon_p = [\varepsilon_{zz} \varepsilon_{zt} \varepsilon_{zt} 2\varepsilon_{zt} 2\varepsilon_{zt} 2\varepsilon_{zt}]^T.
\]

Assuming isotropic elasticity, the constitutive equation for the stress reads

\[
\sigma = \mathbf{C}\varepsilon
\]

with

\[
\mathbf{C} = \frac{E}{(1 + v)(1 - 2v)} \begin{pmatrix}
1 - v & v & v & 0 & 0 & 0 \\
v & 1 - v & v & 0 & 0 & 0 \\
v & v & 1 - v & 0 & 0 & 0 \\
0 & 0 & 0 & 1/2 - v & 0 & 0 \\
0 & 0 & 0 & 0 & 1/2 - v & 0 \\
0 & 0 & 0 & 0 & 0 & 1/2 - v
\end{pmatrix}.
\]

3.1.1. Yield surface

The von Mises yield surface is adopted to describe the boundary of the elastic domain in stress space,
Fig. 7. Measured force–displacement curves for (a) notched tension, (b) tension with a central hole, and (c) double shear experiments with different material orientations ($0^\circ$ = along $z$-direction, $90^\circ$ = along $s$-direction).
\( f(\mathbf{\sigma}) = \mathbf{\sigma} - k = 0 \) \hspace{1cm} (6)

with the von Mises stress \( \mathbf{\sigma} \) and the flow resistance \( k \).

3.1.2. Non-associated flow rule

A non-associated anisotropic Hill’48 flow rule is employed to account for the significant loading direction dependency of the Lankford ratios (see Table 1). In accordance with the plasticity model proposed by Mohr et al. (2010), the flow rule is formally written as

\[
d\mathbf{\varepsilon}_p = \bar{d} \frac{\partial \phi}{\partial \mathbf{\sigma}}
\]

with the plastic multiplier \( \bar{d} \geq 0 \) and the plastic flow potential.
\[
g = \sqrt{G\sigma - \sigma}
\]  
which is defined through the semi positive definite 6 × 6 matrix
\[
G = \begin{bmatrix}
1 & G_{12} & -(1 + G_{12}) & 0 & 0 & 0 \\
G_{12} & G_{22} & -(G_{22} + G_{12}) & 0 & 0 & 0 \\
-(1 + G_{12}) & -(G_{22} + G_{12}) & 1 + 2G_{12} + G_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 3
\end{bmatrix}
\]  

3.2. Self-similar (isotropic) hardening law

The isotropic hardening law defines the evolution of the deformation resistance as a function of the equivalent plastic strain,
\[
k = k[\varepsilon_p]
\](10)  

According to Sung et al. (2010) and Mohr and Marcadet (2015), a weighted combination of the Swift and Voce approximations is used as parametric form,
\[
k[\varepsilon_p] = wk_S[\varepsilon_p] + (1 - w)k_V[\varepsilon_p]
\]  
with
\[
k_S[\varepsilon_p] = A(\varepsilon_p + \varepsilon_0)^n
\]  
and
\[
k_V[\varepsilon_p] = k_0 + Q(1 - e^{-\beta\varepsilon_p}).
\]  

In sum, the parametric form thus features the Swift parameters \(A, \varepsilon_0, n\), the Voce parameters \(k_0, Q, \beta\), and the weighting parameter \(w \in [0,1]\).

3.3. Fracture modeling

3.3.1. Probabilistic fracture model

It is well established that the initiation of ductile fracture in metals depends on the stress state (e.g. Brünig et al. (2008), Bai and Wierzbicki (2008), Sun et al. (2009), Gruben et al. (2011), Chung et al. (2011), Lecarme et al. (2011), Khan and Liu (2012), Luo et al. (2012), Huespe et al. (2012), Malcher et al. (2012), Lou et al. (2014)). Here, we characterize the state of stress through the stress triaxiality \(\eta\), i.e. the ratio of the mean stress \(\sigma_m\) and the von Mises equivalent stress,
\[
\eta = \frac{\sigma_m}{\sigma}.
\]  

The second measure of stress state (characterizing the effect of the third deviatoric stress tensor invariant \(J_3\)) is the Lode angle parameter,
\[
\overline{\theta} = 1 - \frac{2}{\pi} \arccos \left[ \frac{27}{2} \frac{J_3}{\sigma^2} \right].
\]  

Inspired by the work of Teng et al. (2009), a probabilistic fracture initiation model is developed. The deterministic Hosford–Coulomb model proposed by Mohr and Marcadet (2015) predicts the equivalent plastic strain to fracture as a function of the stress state. According to the Hosford–Coulomb model, fracture occurs at an equivalent plastic strain of \(\tau_f\) when the integral condition
\[
\int_0^{\tau_f} \frac{d\varepsilon_{pl}}{\tau_f[\eta, \overline{\theta}]} = 1
\]  


156
is fulfilled. In (19), the analytical expression for the strain to fracture for proportional loading, \( \varepsilon_f^p \), is derived from the Hosford–Coulomb model in stress space (Mohand and Marcadet, 2015),

\[
\varepsilon_f^p[\eta, \theta] = b(1 + c)(\frac{1}{2}((f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_1 - f_3)^2))^\frac{1}{2} + c(2\eta + f_1 + f_3)^\frac{1}{2}
\]  

\[\text{(17)}\]

with the Lode angle parameter dependent trigonometric functions

\[
f_1[\theta] = \frac{2}{3}\cos[\frac{\pi}{6}(1 - \theta)], \quad f_2[\theta] = \frac{2}{3}\cos[\frac{\pi}{6}(3 + \theta)] \quad \text{and} \quad f_3[\theta] = -\frac{2}{3}\cos[\frac{\pi}{6}(1 + \theta)].
\]

The main model parameters are \(a, b, c\), The Hosford exponent \(a\) controls the effect of the Lode parameter, while the friction coefficient \(c\) primarily controls the effect of the stress triaxiality on the strain to fracture. The model parameter \(b\) is a multiplier controlling the overall magnitude of the strain to fracture. It is defined such that it is equal to the strain to fracture for uniaxial tension (which is the same as that for equi-biaxial tension). \(^1\)

To account for the probabilistic nature of the results obtained from fracture experiments on the ALM-made Ti–6Al–4V, we make use of the normal distribution to describe the stochastic behavior of the fracture strain \( \varepsilon_f \). Defining \( p^* \) as the probability of the fracture strain \( \varepsilon_f \) being greater than the value \( \varepsilon_f^p \), the Gaussian model provides the relationship

\[
p^* = p[\varepsilon_f \geq \varepsilon_f^p] = 1 - \frac{1}{2}
\left[ 1 + \text{erf}(\frac{\varepsilon_f^p - \mu}{\sqrt{2} \sigma}) \right]
\]

\[\text{(18)}\]

with the Gaussian parameters \( \mu \) (mean), \( \sigma \) (standard deviation), and the error function

\[
\text{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt.
\]

The inverse relationship reads

\[
\varepsilon_f^p = \mu \varepsilon_f + \sqrt{2} \sigma \text{erf}^{-1}[2(1 - p^*) - 1].
\]

\[\text{(19)}\]

It is assumed that the mean depends on the stress state, while the standard deviation shall be independent of the stress state. Using Eq. (20), the probabilistic fracture model is then rewritten as

\[
\varepsilon_f^p[\eta, \theta, p^*] = \varepsilon_f^p[\eta, \theta] + \sqrt{2} \sigma \text{erf}^{-1}[2(1 - p^*) - 1]
\]

\[\text{(21)}\]

In sum, we have the model parameters \(a, b, c\) that describe the stress state dependency of the mean strain to fracture, and the standard deviation \(b_s\) that describes a shift of the fracture envelope for proportional loading as a function of the probability \(p^*\). For completeness, it is noted that \( \varepsilon_f^p[\eta, \theta] \) must be substituted in Eq. (19) by \( \varepsilon_f^p[\eta, \theta, p^*] \) to make use of the probabilistic model for non-linear loading paths.

3.4. Numerical implementation

The plasticity and fracture initiation models are implemented into the finite element software Abaqus/explicit using the user material model subroutine interface. A basic return mapping scheme with an Euler backward time integration scheme is used to solve the constitutive equations describing the elasto-plastic material response, closely following the procedures outlined in Simo and Hughes (1998).

For a given state at time step \(t_n\) in the finite element software and a known total strain increment \(\Delta \varepsilon = \varepsilon_{n+1} - \varepsilon_{n}\), the algorithm computes the stresses \(\sigma_{n+1}\), the state variables and the equivalent plastic strain \(\varepsilon_{pl}^{n+1}\) at \(t_n+1 = t_n + \Delta t\) with the relationship

\[
\sigma_{n+1} = C^{el}(\varepsilon_{n+1} - \varepsilon_{pl}^{n+1}) = \sigma^{tr} - C^{el}(\Delta \varepsilon^{pl}),
\]

\[\text{(22)}\]

with \(C\) being the elastic stiffness tensor. Whenever the elastic trial stress

\[\text{1} \text{ The parameter } n \text{ has only little effect on the calibrated model as a similar effect can also be achieved by adjusting the coefficients } a \text{ and } b. \text{ It originates from a power law type of isotropic hardening model. We chose } n = 0.1 \text{ as it is a typical hardening exponent for metals. However, it is reemphasized that a different value could also be chosen without changing the reported results.}
lies outside the yield surface, a return mapping scheme based on a backward-Euler integration is used to solve for the derivatives with respect to the plastic multiplier $\lambda$. A Taylor series truncated at the first order element is used to develop $f_n + 1$:

$$f_{n+1}[\sigma_n, k_n] = f_n[\sigma_n, k_n] + \Delta \lambda \cdot \frac{df[\sigma_n, k_n]}{d\lambda} \bigg|^{n+1}_{n}$$

The equation $f_{n+1}[\sigma_n, k_n] = 0$ is then solved for small time steps using an iterative Newton–Raphson method with $i$ being the number of iterations used to find the solution.

$$\Delta \lambda_{n+1} = \Delta \lambda_n - \frac{f_{n+1}[\Delta \lambda]}{df_{n+1}/d\lambda}$$

To initialize the algorithm at the beginning of each time increment, the trial stress and the values of the previous step are utilized. It follows from Eq. (7) that

$$r_{n+1,i}^{pl} = r_{n,i}^{pl} + \Delta \lambda_{n+1} \frac{\partial r_{n,i}^{pl}/\partial \lambda}{\partial \lambda} \bigg|^{n+1}_{n} = r_{n,i}^{pl} + \Delta \lambda_{n+1} \frac{g[\sigma_n]}{\sigma_{nM}} \bigg|^{n+1}_{n}$$

After solving Eq. (25) for $\Delta \lambda_{i+1}$, the stresses and the equivalent plastic strain are updated:

$$\sigma_{i+1} = \sigma - C_{eff} \frac{\partial g[\sigma]}{\partial \sigma} \bigg|^{n+1}_{n} \Delta \lambda_{i+1}$$

$$r_{n+1,i}^{pl} = r_{n,i}^{pl} + \Delta \lambda_{i+1} \frac{\partial r_{n,i}^{pl}/\partial \lambda}{\partial \lambda} \bigg|^{n+1}_{n} = r_{n,i}^{pl} + \Delta \lambda_{i+1} \frac{g[\sigma]}{\sigma_{nM}} \bigg|^{n+1}_{n}$$

The iterative procedure is continued until the convergence criterion

$$|f_{i+1}^{n+1}[\sigma, k]| = |r_{i+1}^{n+1} - k_{i+1}^{n+1}| < TOL$$

is met for $TOL = \sigma_{crit} \cdot 1e^{-4}$.

The probabilistic fracture model is implemented into the same subroutine. If failure is detected, the element deletion flag is set, leading to removal of the corresponding integration point in the subsequent time step (Abaqus, 2014).

4. Model calibration and validation

A combined analytical and numerical approach is taken to identify the model parameters. The finite element models are therefore presented first before detailing the calibration procedures and comparing the simulation predictions with the experiments.

4.1. Finite element models

Finite element simulations are performed for the notched tension, central hole tension and shear experiments using the software Abaqus/explicit. For each specimen, the corresponding finite element model (Fig. 8) comprised only the gage section and a small portion of the specimen shoulders which was located between the global extensometer measurement points shown as blue solid dots in Fig. 4b−d. In all models, we assumed symmetry of the mechanical fields with respect to the specimen mid-plane (x-y-plane), i.e. only half the thickness was modeled with eight first-order elements along the (half-) thickness direction for the NT, the CH and the SH specimens as recommended by Dunand and Mohr (2010). Furthermore, we imposed symmetry boundary conditions along the longitudinal plane (y-z-plane) for all specimens. In case of the NT and CH specimen, an additional symmetry condition could be imposed along the x−z-plane at the specimen center.

The axial displacement is applied to the mesh boundaries using a smooth velocity ramp with zero slope at $t = 0$ (i.e. zero initial acceleration). The stable explicit time step has been automatically adjusted through mass scaling such that each simulation completed after approximately 100,000 time steps. Analogously to the experiments, we also extracted the local relative displacement between two central points on the specimen surface (see red dots in Fig. 4) to compute the axial strain at the center.
Fig. 8. Finite element meshes used for (a) central hole tension, (b) notched tension, and (c) shear loading.
4.2. Plasticity model parameter calibration

The parameters \( \{G_{12}, G_{22}, G_{33}\} \) defining the anisotropic flow potential are directly linked to the measured Lankford ratios,

\[
G_{12} = -\frac{G_{12}}{1 + G_{12}}, \quad G_{22} = -\frac{G_{12}}{G_{22} + G_{12}}, \quad \text{and} \quad r_{45} = \frac{1}{2} \left( \frac{G_{33}}{1 + G_{22} + 2G_{12}} - 1 \right),
\]

and can thus be explicitly calculated based on the average values for \( r_x \), \( r_z \) and \( r_{45} \), given in Table 1:

\[
G_{12} = \frac{<r_x>}{1 + <r_x>}, \quad G_{22} = \frac{<r_x>}{1 + <r_x>}, \quad \text{and} \quad G_{33} = \frac{1 + 2 <r_{45}>}{<r_x> + <r_z>}.
\]

The parameters of the isotropic hardening law are identified in a two-step procedure. Firstly, we calculated the average true stress (from all \( N = 12 \) uniaxial experiments) for a given logarithmic plastic strain \( \varepsilon_p^{(i)} \),

\[
<\sigma^{(i)}> = \frac{1}{N} \sum_{j=1}^{N} \sigma^{(j)} [\varepsilon_p^{(j)}]
\]

Subsequently, the Swift parameters \( \{A, \varepsilon_0, n\} \) and the Voce parameters \( \{k_0, Q, \beta\} \) are determined from two separate fits to the average stress strain curve up to a strain of 0.1 (Fig. 9a). Note that the difference between the Swift and Voce approximations only becomes significant at large strains (> 0.1).

4.3. Plasticity model validation

Fig. 11 provides the finite element model predictions for nine different experiments with a force-displacement response close to the average of the corresponding type of experiment (see Fig. 7 for material-related randomness in the specimen response). In addition to the calibration experiment (Fig. 11d), the model predictions agree reasonably well with the experimental results, both as far as the global force-displacement and local strain evolutions are concerned. The least accurate prediction is observed for shear loading. The force is underestimated by more than 10% for the SH0 and SH90 experiments which is not only attributed to the scatter in the material response, but also to shortcomings in the plasticity model formulation. Note that the principal stresses are aligned with the s- and z-axes in all experiments except for SH0 and SH90. In the SH45 experiment, the material is subject to tension along the z-direction and compression along the s-direction in the left gage section, while the stresses in the second gage section feature opposite signs. The reasonable agreement of the prediction for SH45 is therefore seen as partial support of the model assumption of a J3-independent yield surface.

4.4. Loading paths to fracture

The loading paths to fracture, i.e. the evolution of the equivalent plastic strain as a function of the stress state, will serve as basis for calibrating the fracture model parameters. For each NT, CH and SH experiment performed, the evolution \( \{\eta[t], \mathbf{n}[t]\} \) of the stress state and the equivalent plastic strain \( \varepsilon_p[t] \) must be extracted from the corresponding finite element simulation. For this, it is assumed that fracture initiated at the location within the gage section where the equivalent plastic strain is the highest at the instant when the applied displacement (in the simulation) equates to the displacement to fracture measured experimentally (see third column in Tables 2 and 3). This point is located on the surface, near the gage section center for the SH specimens (See Fig. 10c). For the CH and NT specimens, this location (highlighted in Fig. 10a and b) typically corresponds to the intersection point of the specimen’s axial plane of symmetry with the free curved boundary (outer radius of NT specimens, central hole boundary for all CH specimens). In NT specimens with high displacement to fracture, through thickness necking shifted the point of highest straining to the specimen center (Fig. 12).

Except for the NT experiments with pronounced necking, the stress state remained more or less constant and the plane stress assumption held approximately true. Fig. 13 shows the evolution of the equivalent plastic strain as a function of the applied displacement and the stress triaxiality for the NT, CH and SH experiments. A plot of the evolution of the Lode angle parameter is omitted due to its known dependence on the stress triaxiality under plane stress conditions. For tension with a central hole, the stress triaxiality is close to the theoretical value for uniaxial tension (\( \eta = 0.33 \)). In the SH specimens, the theoretical value for pure shear (\( \eta = 0 \)) is approximately achieved. In the NT specimens with fracture initiation near the notch
root, approximately uniaxial conditions prevailed. In all other NT experiments, the stress state at the specimen center increases from a stress triaxiality of about 0.5 to values as high as 0.7 after the instant of fracture.

Single element simulations are performed to identify the stress state and the equivalent plastic strain based on the surface strain measurements from the V-bending and punch experiments. Note that the stress state remained constant in these experiments (absence of necking). Tables 2 and 3 include all hybrid numerical-experimental results. To visualize the effect of stress state, the strain to fracture is also plotted in Fig. 14a as a function of the stress triaxiality for all experiments with more or less constant stress state (note that such plot might lose its meaning when including data from experiments with significant variations in stress state prior to loading).

4.5. Fracture model calibration

The experimental data shown in Fig. 14a is used to identify the fracture model parameters \{a, b, c, \beta\}. Due to the stress state independence of the noise in Eq. (20), the parameters \{a, b, c\} can be determined from the dependency of the mean value

![Graph](image-url)

Table 4

<table>
<thead>
<tr>
<th>G_{12} [-]</th>
<th>G_{22} [-]</th>
<th>G_{33} [-]</th>
<th>A [MPa]</th>
<th>\epsilon_0 [-]</th>
<th>n [-]</th>
<th>Q [MPa]</th>
<th>\beta [-]</th>
<th>k_0 [MPa]</th>
<th>w [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.37</td>
<td>1.10</td>
<td>4.75</td>
<td>1507.6</td>
<td>0.037</td>
<td>0.150</td>
<td>207.4</td>
<td>13.947</td>
<td>907.3</td>
<td>0.66</td>
</tr>
<tr>
<td>-0.45</td>
<td>0.83</td>
<td>3.78</td>
<td>1424.6</td>
<td>0.042</td>
<td>0.137</td>
<td>237.0</td>
<td>5.555</td>
<td>953.6</td>
<td>0.99</td>
</tr>
</tbody>
</table>
of the strain to fracture on the stress triaxiality. We thus define bins on the stress triaxiality axis within which the stress triaxiality variations are assumed to be negligible:

- The first bin for pure shear (average stress triaxiality of \(<\eta>_I = -0.05\)) includes all \(N = 17\) black dots and features an average strain to fracture of

\[
<\varepsilon_f>_I = \frac{1}{N} \sum_{i=1}^{N} \varepsilon_f^{(i)} = 0.42.
\]

- The second bin for uniaxial tension (average stress triaxiality of \(<\eta>_II = 0.34\)) includes all \(M = 15\) blue dots and features an average strain to fracture of

\[
<\varepsilon_f>_II = \frac{1}{M} \sum_{i=N+1}^{N+M} \varepsilon_f^{(i)} = 0.26.
\]

Fig. 10. Deformed finite element meshes for (a) NT45, (b) CH0 and (c) SH90 showing the distribution of the equivalent plastic strain at the instant of fracture initiation for the NT45_2, CH0_2 and SH90_5 experiments, respectively. Due to the emergence of a shear band, the simulation results of the SH experiments have been reconfirmed for different mesh sizes.
The third bin for biaxial tension (average stress triaxiality of $\langle \eta \rangle_{\text{III}} = 0.52$) includes all $L = 5$ red dots and features an average strain to fracture of

$$\langle \varepsilon_f \rangle_{\text{III}} = 1.022$$

An exact fit of Eq. (20) to the average values (using the “fminsearch” function of Matlab) yields the parameters $a = 1.7$, $b = 0.26$ and $c = 0.06$. The parameter $b_s$ is then estimated using a maximum likelihood approach. Considering the strains to

Fig. 11. Validation of the finite element model for different experiment types (NT, CH and SH) and different specimen orientations. Experimental results are shown as dotted lines, and the simulation results are shown as solid lines.
fracture $\varepsilon_f^{(1)} \ldots \varepsilon_f^{(N+M+L)}$ as independent random variables, it can be shown that the Maximum Likelihood Estimate (MLE) of $b_s$ equals the standard deviation of the mean shifted pooled sample, i.e.

$$b_s = \sqrt{\frac{1}{N+M+L} \left( \sum_{i=1}^{N} \left( \varepsilon_f^{(i)} - \langle \varepsilon_f \rangle_1 \right)^2 + \sum_{i=N+1}^{N+M} \left( \varepsilon_f^{(i)} - \langle \varepsilon_f \rangle_2 \right)^2 + \sum_{i=N+M+1}^{N+M+L} \left( \varepsilon_f^{(i)} - \langle \varepsilon_f \rangle_3 \right)^2 \right)}$$

which yields $b_s = 0.08$. The corresponding cumulative probabilities for the strains to fracture for pure shear (first bin) and uniaxial tension (second bin) are shown in Fig. 14b next to the experimental data. The data for biaxial tension is not shown because of the small number of data points in that bin.

Using the calibrated probabilistic fracture initiation model, we plotted the “fracture envelope” for proportional plane stress loading in Fig. 14a. The solid black line shows the envelope for $p^* = 50\%$; according to the Gaussian distribution, it describes the average material response. The dashed black line describes the model response for $p^* = 90\%$, i.e. with 90% probability the strain to fracture for a given stress state is higher than that given by the dashed envelope.

5. Discussion

The particular feature of the current Ti–6Al–4V material is the organization of the microstructures in domains that are due to prior beta grains. This causes macroscopic material heterogeneity at the millimeter scale in addition to the conventional microscopic heterogeneity of a polycrystalline material. To shed more light on the effect of the macroscopic heterogeneity on the material response, the DIC measured surface strain fields are analyzed. In addition, we also performed selected fracture experiments on a Ti–6Al–4V sheet stock without any prior beta grain structure.

5.1. Effect of macroscopic material heterogeneity

Fig. 15 shows the surface strain fields (axial component of the logarithmic strain tensor) at different stages throughout the loading of the UT specimens up to the point of specimen fracture. The color bar is adjusted such that the red color corresponds to the maximum strain value given above each plot. The main observation is that the strain fields are heterogeneous at the scale of the specimens. Moreover, domains of more or less homogeneous strain distributions are observed with a morphology...
reminiscent of that of prior-beta grains. Note that the orientation of the homogeneous strain domains changes as a function of the specimen orientation which is another evidence of a correlation with the prior beta grain morphology (e.g. compare Fig. 15c with Fig. 2b). The differences in the deformation resistance of neighboring prior-beta grains already becomes apparent at small strains (see leftmost plots in Fig. 15) and leads to an early localization of nearly all deformation in the weakest prior beta grain where fracture initiates.

Different from the UT specimens, the CH and NT specimens feature strong gradients in the mechanical fields even for homogeneous materials (see Fig. 10a and b). In the CH specimens, the equivalent plastic strain decreases from its maximum $\tau_f$ to $\tau_f/2$ over a distance of 1.1 mm. This distance is 3.6 mm in the case of the NT specimens. It appears from the surface strain

![Graphs showing mechanical fields with displacement and triaxiality](https://via.placeholder.com/150)

**Fig. 13.** FEA prediction of the relationship between the equivalent plastic strain and the applied displacement (left column plots) and the evolution of the stress triaxiality (right column plots) for all notched tension, tension with central hole and shear experiments. The solid dots indicate the instants of fracture initiation in the experiments.
fields for CH specimens that the gradient in the applied mechanical fields is able to overwrite the effect of material heterogeneity at the millimeter scale, i.e. the location of strain localization and fracture seems to be controlled by the mechanical fields (Fig. 16) to an extend that the strain fields preserve the symmetries of the initial specimen geometry. The effect of material heterogeneities then only becomes apparent after fracture initiation. The cracks seem to initiate and propagate along the prior beta grain boundaries which results in asymmetric specimen fracture: firstly, the fracture does not initiate simultaneously in the left and right hole ligaments; secondly, the crack paths in the plane of the specimen are inclined with respect to the horizontal axis which is remarkably different from those for macroscopically homogeneous materials (e.g. DP steels tested by Roth and Mohr, 2014). In the NT specimens, the gradients in the mechanical fields are not strong enough to overwrite the effect of material heterogeneity. The DIC surface strain fields do not respect the planes of symmetry of the specimen (Fig. 17). The contour plots for NT45 and NT90 show that already at an early stage of loading, the deformation localizes above and below the narrowest

![Image](image_url)

**Fig. 14.** (a) Hosford–Coulomb fracture envelope for a fracture initiation probability of 90% (dashed line) and 50% (solid line); the dashed green line shows the identified envelope for the sheet material; (b) Fit of normal distribution (solid lines) to experimental data (solid dots) from Tables 2 and 3 in the vicinity of pure shear (black) and uniaxial tension (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
cross-section. As for the UT specimens, fracture eventually initiates in the zones of localization that are already defined at an early stage of plastic loading.

Fractographic analysis confirms the above observations. Fig. 18 shows a micrograph of a crack that initiated in an CH0 experiment. Recall that the dark horizontal lines in Fig. 18a are associated with the progressive layer-by-layer build-up of the material while the weak gray contrast is due to different prior beta grain orientations. The crack seems to have initiated near the hole boundary (response dominated by applied mechanical gradients). Then, the crack traveled from the hole boundary towards the left, before hitting and following a prior beta grain boundary towards the vertical direction (response dominated by material heterogeneity). The zoom in Fig. 18b highlights the characteristic microstructure of a prior beta grain boundary at the corner where the crack deflected. Another micrograph of a crack running along a prior beta grain boundary is shown in Fig. 18c. The prior beta grain boundary is clearly visible with the crack passing through highly sheared alpha lamellae.

Fig. 18. Logarithmic axial strain fields in uniaxial tension experiments: (a) along the build-up direction, (b) along the diagonal direction, (c) along the deposition direction.
5.2. Comparison with prior beta-grain free Ti-6Al-4V sheet stock

A comprehensive plasticity and fracture testing program is also carried out on a 1.5 mm thick Ti-6Al-4V sheet (purchased from McMaster Carr) with the same chemical composition as the ALM alloy. The sheet microstructure (Fig. 19) is composed of 5 μm diameter equi-axed alpha grains and beta grains with α lamellae inside. However, unlike the ALM material, no trace of prior β grains is found.

Using the same types of specimens and following the same experimental procedures as for the ALM material, the plasticity and fracture properties of the Ti-6Al-4V stock are determined. The stress-strain curves for the rolling and transverse sheet directions lie on top of each other, while that for the 45° directions lies slightly below, showing less strain hardening (Fig. 20a). Note that all curves fall within the group of curves determined for the ALM material (Fig. 6a). The corresponding Lankford ratios are: $r_0 = 0.80$, $r_{45} = 1.12$, and $r_{90} = 1.52$. The dotted lines in Fig. 20b–d show the measured force-displacement curves for the fracture specimens extracted from sheet material in the rolling direction. The comparison with the results for the ALM material reveals that the displacement to fracture of the sheet stock is higher. After calibrating the above plasticity model parameters for the sheet stock (see last line of Table 4), the loading paths to fracture have been extracted from the finite

![Logarithmic axial strain fields in central hole tension experiments: (a) along the build-up direction, (b) along the diagonal direction, (c) along the deposition direction.](image)
element simulation of each fracture experiment, and used to identify the deterministic Hosford–Coulomb model parameters ($a = 1.18$, $b = 0.89$ and $c = 0.06$). In Fig. 14a, the obtained fracture envelope (green dashed line) is shown next to the fracture data for the ALM material. The comparison clearly shows that the “conventional” sheet stock of the Ti–6Al–4V provides a significantly higher ductility than the ALM material with an estimated strain to fracture for uniaxial tension of 0.89 (versus a median of 0.26 for the ALM material). The differences are less significant for pure shear (0.57 versus 0.41), but still almost 50% higher on average.

6. Conclusions

A comprehensive experimental program is performed to characterize the plasticity and fracture response of Ti–6Al–4V components made through Additive Layer Manufacturing (ALM). A wire-feed process has been used instead of a powder-based technique to reduce the risk of contaminations during manufacturing. For reference, we also performed all experiments on conventional Ti–6Al–4V sheets made through casting followed by rolling. While the experimental results for the sheet stock showed a high repeatability, the results for the ALM material showed significant scatter. The standard deviation for the strains to fracture was 0.08 which corresponded to relative standard deviation of 0.08/0.42 = 19% and 0.08/0.26 = 31% for pure shear and uniaxial tension, respectively. A simple probabilistic model is proposed to describe the fracture response of the ALM material by incorporating a Gaussian probability density function into the Hosford–Coulomb fracture initiation model. The deterministic response of sheet material falls well within the population of stress-strain curves for uniaxial tension measured for the ALM material. However, the fracture strains of the sheet material are

![Fig. 17. Logarithmic axial strain fields in notched tension experiments: (a) along the build-up direction, (b) along the diagonal direction, (c) along the deposition direction.](image-url)
significantly higher. For example, for a stress triaxiality of 0.33 (uniaxial tension), the calibrated models suggest that the strain to fracture for the sheet material is more than three times higher than the corresponding average value for the ALM material.

Microscopic analysis showed that the organization of the ALM microstructure in macro-domains (prior beta grains) of statistically homogenous texture affects the fracture response. In experiments with weak mechanical gradients, the surface strain fields show that the strain distribution becomes inhomogeneous at the millimeter scale due to the localization of deformation in individual prior beta grains. Furthermore, the observed crack paths follow the prior-beta grain boundaries. From a manufacturing point of view, it is worth noting that the layered material deposition did not leave any noticeable trace in SEM observations of the material microstructure. However, the growth of the prior beta grains follows the thermal gradient during solidification which is related to the welding torch during wire deposition.

Fig. 18. Crack path deflection at prior β-grain boundary: (a) fracture of the left ligament of a CH0 specimen; (b) SEM micrograph of the zone highlighted in (a). (c) SEM micrograph of highly-sheared α-lamellae along the crack path.
**Fig. 19.** SEM micrograph of the Ti–6Al–4V stock sheet with equiaxed grains.

**Fig. 20.** Results for the Ti–6Al–4V sheet material: (a) true stress-strain curves for different in-plane specimen orientations; force-displacement responses for different fracture specimens extracted along the rolling direction: (b) notched tension, (c) central hole tension, and (d) shear.
Acknowledgments

The authors would like to thank Alexandre Tanguy and Dr. Eva Heripré (LMS — Ecole Polytechnique) for their help with the microscopic analysis. Thanks are also due to Professor Tomasz Wierzbicki (MIT) for valuable discussions. The partial financial support through the MIT Industrial Fracture Consortium and the CNRS is gratefully acknowledged.

References

Tersing, H., Lorentzon, J., Francois, A., Lundb...